TEXTBOOK No. 7004

Fundamentals of Precision Measurement

TEXTBOOK

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CONTENTS

1. PREFACE

In modern industries, the uniformity of parts (which was not needed in the handicraft industries) is a vital factor, and for this reason, the technology of precision measurement has been developed. In modern times, sophisticated articles such as airplanes, automobiles, ships, etc. which consist of tens (or hundreds) of thousand parts are mass produced and these parts are manufactured in different factories. This necessitates the standardization of the parts for interchangeability, and optical and electrical (as well as mechanical) measuring instruments are used. In these days, a big progress is made in the measuring technology to replace analog measurement with digital measurement. Measurement data is analyzed upon finishing measurement and is immediately fed back to the manufacturing process.

In this pamphlet, the fundamentals of precision measurement which are necessary for inspecting the dimensions of the parts manufactured will be explained.

2. WHAT IS PRECISION MEASUREMENT ?

According to the description of JIS, the precision measurement is the measurement performed with high accuracy. Precision machining means accurate cutting or grinding of a workpiece to the specified dimensions and not minute machining. For example, a gauge block has a flat face (with high flatness and parallelism) and its tolerance of dimensions is less than 1μm. It is very simple in shape; however, it is accurately machined.

Measuring length, mass, time span, temperature, etc. is usually performed in our daily life and is indispensable for us; and measurement is defined as follows.

When a quantity is to be measured, a certain amount of quantity of the same kind (to which a name is given) is defined and, the former is represented by how many times or fractions of the latter does the former consist of. The certain amount of quantity named is called unit and it must indicate a constant value in specified ambient conditions. Ordinarily, three basic units of length, mass, and time are used. In the present training course, only the precision measurement of length will be explained.

3. UNIT AND STANDARD OF LENGTH

3.1 Units of Length in Ancient Ages

There have been two types of systems in measuring length in the history of mankind; one in the Orient, and the other in the Occident. In the Orient, Sino-Japanese system is originated from the areas along the Hwang Ho (Yellow River) and Indus river. In the Occident, on the other hand, English system is originated from the civilization developed along the Nile River, Tigris River, and Euphrates River (4000 B.C.).

In the ancient ages, the lengths of the portions of a human body seemed to be used as the units of length as shown below.

(1) Foot . . . . foot
(2) Duim . . . . . . thumb
(3) Finger . . . . . . finger
(4) Pouse . . . . . . thumb

In the Babylonian age, the following units were used.

(5) 1 millia = 60 studies
(6) 2 studies = 60 x 12 cupits
(7) 1 cupit = 30 digits

ⓐ Cupit means elbow and is about 500mm.
ⓑ Study is the distance over which a man walks in an ordinary speed from the time the sun begins to rise to the time it completely appears above the horizon (about two minutes). One study is 185m to 195m.

In the Orient, the length of the flute in the Oshiki era (1000 B.C.) in China was used as the standard; and decimal system was employed for representing the units of length as follows.

(8) Shaku = 10 Sungs (length of flute: 90 particles of millet)
(9) Sung = 10 minutes (the length of a particle of millet is a minute)

3.2 Conversion to Metric System

In the ancient ages, countries had their own units of length and the values of length measured in a country could not be compared with those measured in the other countries. As progress had been made in technology and, trade between different countries had become flourishing, the different units had to be imminently unified.

In 1664, Huygens (Dutch physicist) thought of using the period of the swing (which had been discovered to be constant by Galileo Galilei) as the standard of length. However, the period was affected by the mass of the string, the position of the center of the mass of the ball, **air dragging**, and the **abrasion of string.** These factors attenuated the reciprocating motion of the swing. Consequently, his idea could not be put into practice.

In 1670, Mouton (French schloar) proposed to use one-ten millionth of quadrant (which is equal to one-forty millionth of meridian) as the standard of length. Since then, the measuring method of the earth had been searched. In 1791, the committee appointed by the French government determined to call one-ten millionth of quadrant from the north pole to the equator "meter" as the new unit of length. It took 120 years from proposal to introduction.

Measurement began in June, 1792. The distance between Dunkerque in nothern France and Barcelona (in Spain) on the Mediterranean sea was measured by triangulation. A reversible type **goniometer** was used at that time. It was invented by Borda and its accuracy was as high as one second. The measurement ended in June, 1798.

The scale called "toharz" in France was used for the measurement and the distance between the north pole and equator was determined to be 5,130,740 toharzes. The value was obtained by fully using correction technology available at that time.

In 1799, a bar was made of plutinum which was 1m long and its cross section was 25.3mm x 4mm. "Metre Des Archives" was carved on it.

In 1870, an international conference on length was held in Paris. In May, 1875, seventeen nations signed the international metric system treaty by which the International Bureau of Weights and Measures was founded at Sêpre in the outskirt of Paris. In 1876, they began fabricating the meter prototype and reproducing it for the nations which ratified the treaty.

Thirty-two units of bars comprising 90% of platinum and 10% of iridium were made. They were 1020mm long and X-shaped in cross section. The neutral faces over 8mm in the vicinity of the edges were polished and graduation lines with the thickness of 6 – 8μm were carved on them and the lengths between the lines were made to 1m as closer as possible at 20°C. See Figure 1.

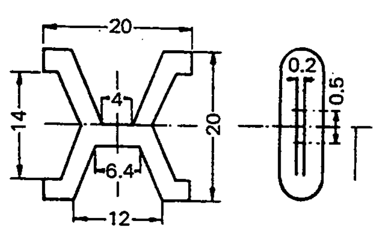


Figure 1. Cross sectional view of meter prototype

Among the 32 meter prototypes, No. 6 meter prototype was determined to be closest to the Metre des Archives and was designated as the international meter prototype in the first International Conference of Weights and Measures held in 1889. It was stored in the International Bureau of Weights and Measures.

In 1889, Japan ratified the international metric system treaty and received No. 22 meter prototype in 1890. It had been Japan's meter prototype until June 30, 1961 and is stored in the NRLM (National Research Laboratory of Metrology).
In 1956, it was sent to Switzerland and new lines are carved on it at the International Bureau of Weights and Measures. Graduation lines with 1mm intervals were carved too and the length was revised to indicate 1m at 20°C.

3.3 Introduction of Light Wave Length for Standardization

Since the graduation lines were not fine enough, the length sculptured on the meter prototype contained an uncertainty which was about 0.2μm. The length might be changed due to aging and the meter prototype might be destroyed by the war (such as World War I 1914 – 1918). Therefore, an effort was made to find an invariable value in natural phenomena which could be used as a new standard replacing the meter prototype. For this purpose, the wave length of a monochromatic light was most universal and reliable, and experiments were performed on the wave length of Cd red light at nine places in the world including Japan. Finally in the seventh International Conference of Weights and Measures held in 1927, 1m was defined as follows.

The wave length of Cd red light

$\lambda CdR = 0.64384696 \mu m.$
$1m = 1553164.13 \lambda CdR$

under the following conditions

Temperature: 15°C (hydrogen thermometer)
Atmospheric pressure: 760mmHg
Ambient: Dried air containing 0.03% of CO_2
(gravitational acceleration g = 980.665cm/s^2).

Thus, the standard length could be easily obtained by a Cd lamp sealed in a cathode ray tube and power lines.

During the period of World War II, a great progress was made in atomic physics and extensive research works were performed on the radiations of isotopes with even atomic numbers which emitted monochromatic lights that were more ideal then those of natural elements. In the International Conference of eights and Measures held in 1957, it was proposed that 1m be equal to 1650763.73 times the wave length (in vacuum) of the light emitted by transition between Kr^{86} $2P_{10}$ and $5d_5$ (Kr^{86} orange light). The proposal was approved in the eleventh Conference held in October, 1960. This decision was proclaimed as a governmental ordinance on July 1, 1961 in Japan.

That is, the following is set forth in the Measurement Law, Article III, Clause I: "The unit of measuring length is meter. One meter is equal to 1650763.73 times the wave length of the light emitted by transition between the energy levels between Kr^{86} 2P10 and $5d_5$. It must be measured in vacuum in compliance with the decision of the International Conference of Weights and Measures which is also described in the governmental ordinance."

By this definition, one meter can be reproduced with accuracy of about 10^{-8} which is 0.01μm.

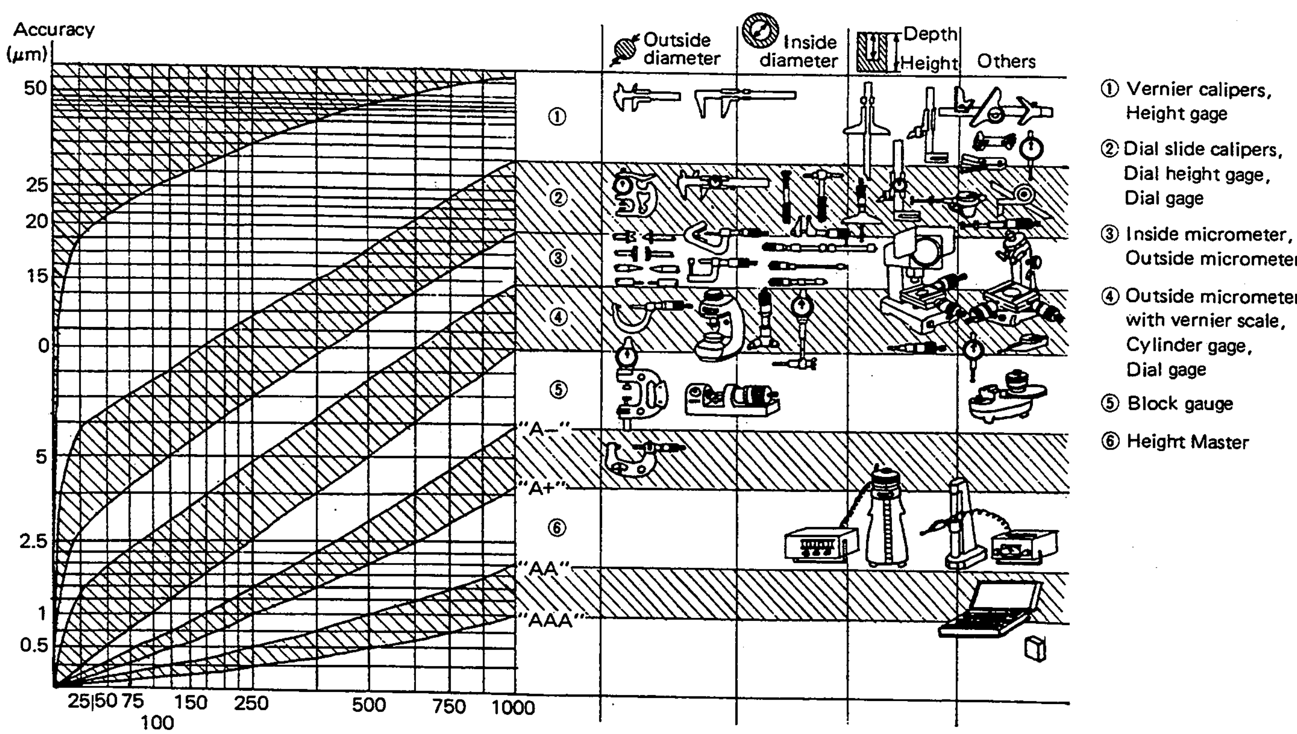


Figure 2. Accuracies of measuring instruments specified by JIS

4. TYPES AND ACCURACIES OF MEASURING INSTRUMENTS

4.1 Selection of Measuring Instruments

When dimensions of a workpiece are to be measured, the accuracy of measurement depends on the measuring instrument selected. For example, if the outer diameter (100mm) of a cast iron product is to be measured, the vernier calipers sufficiently play a role. However, if the diameter of a tap gage with the same diameter is to be measured, even an outside micrometer is not enough in accuracy. An electric or pneumatic micrometer which is more accurate must be used with a gauge block.

Figure 2 illustrates the accuracies of measuring instruments set forth in JIS. With reference to the above specifications in addition to the causes of errors later described, fairly accurate values can be obtained by correction.

It has been recommended that the ratio of the tolerance of a workpiece to the accuracy of a measuring instrument be 10 : 1 in an ideal state and must be 5 :1 in the worst case. Otherwise, the tolerance is mixed with the measurement error; and a good component is diagnosed faulty and vice versa. See Figure 3.

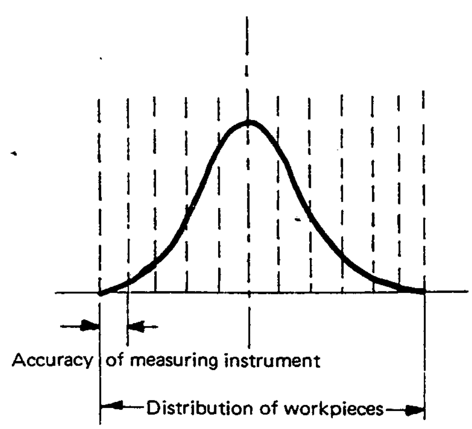


Figure 3.

4.2 Sensitivity and Accuracy of Measuring Instrument

The performance of a measuring instrument is evaluated by its sensitivity and accuracy. **The sensitivity is a degree to which a measuring instrument can sense the variation of the quantity to be measured.** If the magnification ratio of an electric micrometer is increased, one scale interval can be made to represent smaller value, from 1μm to 0.1μm or 0.01μm. However, it may be all but the needle is more responsive and oscillates; and it may not indicate the actual value. Therefore, if the measuring tool is too sensitive, indication becomes unstable and it cannot be used correctly.

Accuracy is the overall correctness of a value obtained by a measuring instrument, in which the bias and dispersion of measured values are the points of concern.

For an outside micrometer, the sensitivities of minimum readings 0.01mm and 0.001mm are 0.01mm and 0.001mm, respectively. In order that it can indicate a correct value with high accuracy, its structure, material, manufacturing process, and function must be pertinent. The flatness and parallelism of the measuring faces, measuring force, bending of the frame, and overall error must be within the specified range.

According to JIS, the overall error of the outisde micrometer must be

$$\pm (1 + L/75)\ \mu m$$

where, L is the maximum measuring length in mm.

4.3 Absolute Measurement and Comparison Measurement

The absolute measurement is defined as follows.

(1) To realize a quantity determined by a definition and to perform measurement using the quantity, or

(2) To obtain the actual value by measuring the workpiece.

When this method is applied, data must be corrected by taking into consideration temperature, elastic deformation, and other conditions to obtain an actual value.

When correct measurement is to be performed in a production line, the above correction is not necessary if comparison method of measurement is used, that is,

(1) Make a standard piece to the specifications of the workpiece to be measured;

(2) Measure the standard piece with high accuracy to obtain a standard value;

(3) Minimize the temperature difference between the workpiece and standard piece, and compare the two for difference.

5. CAUSES OF MEASUREMENT ERRORS AND CORRECTION

5.1 Effect of Temperature

The workpiece expands and contracts as temperature changes. Therefore, a temperature must be designated at which a workpiece must be measured. The temperature is called standard temperature and 20°C has been used as the standard temperature since 1932 in industrialized countries.

However, it is very difficult to keep the temperatures of the workpiece, standard piece, and measuring instrument at 20°C. For example, even is a temperature-controlled room in which the temperature is kept rather stable at 20°C when measured in horizontal plane, temperature distribution is not uniform when measured in vertical direction. In some case, the difference is as large as 1°C per 2m in vertical direction. Therefore, the temperature of the workpiece must be measured accurately and the value must be corrected by means of thermal expansion coefficient.

If temperature changes, the length L of a workpiece varies as follows.

$$\delta L = L \cdot \alpha \cdot \delta t \qquad (1)$$

where

L : original length of the workpiece
α : thermal expansion coefficient
δt : temperature variation

The thermal expansion coefficient of the gauge block is specified by JIS as follows.

$$\alpha = (11.5 \pm 1.0) \times 10^{-6} /\text{deg}$$

The thermal expansion coefficients of several materials are listed in Table 1.

If a rod with thermal expansion coefficient αP is to be measured by comparison with a standard rod with thermal expansion coefficient αN (the length of the latter at 20°C is LN20) under the conditions that their temperatures are as follows:

$tN = 20° + \delta tN$
$tP = 20° + \delta tP,$

the difference between the two is assumed to be
$\delta\ell = \ell P - \ell N.$

Table 1. Thermal expansion coefficients of materials

Material	Thermal expansion coefficient (/°C)	Material	Thermal expansion coefficient (/°C)
Cast iron	9.2~11.8x10⁻⁶	Steel	11.5x10⁻⁶
Carbon steel	11.7−(0.9xc%)x10⁻⁶	Tin	23.0x10⁻⁶
Chromium steel	11~13x10⁻⁶	Zinc	26.7x10⁻⁶
Nickel-chromium steel	13~15x10⁻⁶	Duralumin	22.6x10⁻⁶
Copper	18.5x10⁻⁶	Plutinum	9.0x10⁻⁶
Bronze	17.5x10⁻⁶	Ceramics	3.0x10⁻⁶
Gunmetal	18.0x10⁻⁶	Silver	19.5x10⁻⁶
Aluminum	23.8x10⁻⁶	Crown glass	8.9x10⁻⁶
Brass	18.5x10⁻⁶	Flint glass	7.9x10⁻⁶
Nickel	13.0x10⁻⁶	Quartz	0.5x10⁻⁶
Iron	12.2x10⁻⁶	Vinyl chloride	7~2.5x10⁻⁶
Nickel steel(58% nickel)	12.0x10⁻⁶	Phenol	3~4.5x10⁻⁶
Invar (36% nickel)	1.5x10⁻⁶	Polyethylene	0.5~5.5x10⁻⁶
Gold	14.2x10⁻⁶	Nylon	10~15x10⁻⁶

The difference between the two at 20°C

$\delta\ell 20 = \ell P20 - \ell N20$

is given as follows:

$$\delta\ell 20 = \delta\ell(1-\alpha P\cdot\delta tP) - \ell N20(\alpha P\cdot\delta tP - \alpha N\cdot\delta tN)(1+\alpha N\cdot\delta tN) \qquad (2)$$

This is obtained by modifying

$$\delta\ell 20 = \delta\ell - \ell P\alpha P\alpha P\delta tP + \ell N\alpha N\delta tN$$

which is derived from

$$\ell P20 = \ell P(1-\alpha P\cdot\delta tP)$$
$$\ell N20 = \ell N(1-\alpha N\cdot\delta tN).$$

In the above equation, $\alpha P\cdot\delta tP$ and $\alpha N\cdot\delta tN$ are negligibly small when compared with 1, therefore

$$\delta\ell 20 \doteqdot \delta\ell - \ell N20(\alpha P\cdot\delta tP - \alpha N\cdot\delta tN). \qquad (3)$$

The actual length of the rod at 20°C can be obtained as follows:

$$\ell P20 = \ell N20 + \delta\ell 20.$$

For example, the rod to be measured and the standard rod are as follows:

	Length	Temperature	Thermal expansion coefficient
Rod to be measured	approx. 100	$\delta tP = -0.9°C$	$\alpha P = 18.5 \times 10^{-6}$ (brass)
Standard rod	LN = 100.0012	$\delta tN = +0.8°C$	$\alpha N = 11.5 \times 10^{-6}$ (steel)

If the length of the rod is different from the standard rod by $-61\mu m$,

$$\delta\ell 20 = -0.061 - 100(-18.5\text{x}10^{-6}\text{x}0.9 - 11.5\text{x}10^{-6}\text{x}0.8)\text{mm} = -0.061 - (-0.0026)\text{mm} = -58.4\mu m.$$

The length at 20°C is

$$\ell P20 = 100.0012 - 0.0584 = 99.943\text{mm}.$$

If their temperatures are the same and thermal expansion coefficients αP and αN are different, $\delta tP = \delta tN = \delta t$ in equation (3).

Therefore,

$$\delta\ell 20 = \ell N20(\alpha P - \alpha N)\delta t \qquad (4)$$

If the temperatures are different by δt and αP and αN are the same,

$\delta tP - \delta tN = \delta t$ and $\alpha P = \alpha N = \alpha$ in equation (3).

Therefore,

$$\delta\ell 20 = \delta\ell - \ell N20\cdot\alpha\cdot\delta t. \qquad (5)$$

If $\alpha P = \alpha N$ or $\delta t = 0$ in equations (4) and (5), $\delta\ell 20 = \delta\ell$. That is, if the thermal expansion coefficients and temperatures of the rod to be measured and standard rod are the same, the difference in length obtained at a temperature other than 20°C is the same as the difference at 20°C.

When a test piece is carried in a room where temperature is controlled to 20°C, the variation of temperature in the test piece causes a problem. The bigger are the temperature difference and heat capacity of the test piece, the longer time does it take to reach the temperature of the room.

When iron cylinders (100m long with several dia.) at t_0°C are placed on a wooden table and iron surface plate in a room at t_1°C, their temperatures vary as shown in Figures 4 and 5.

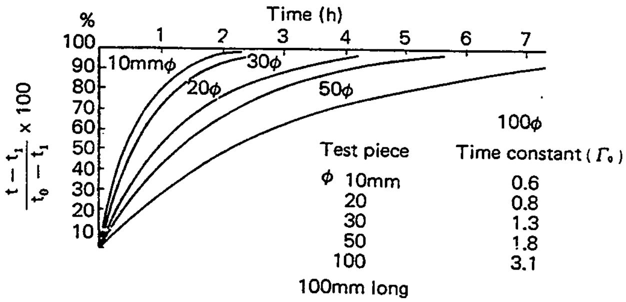


Figure 4. Temperature variations of iron cylinders (on wooden table)

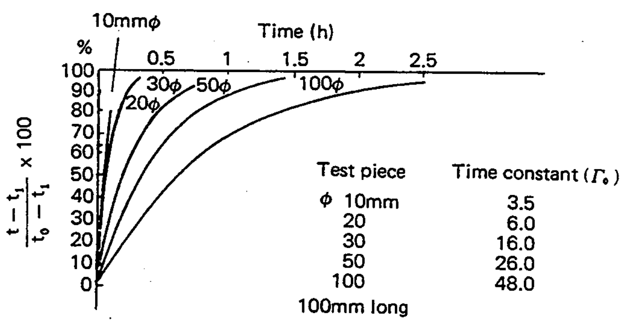


Figure 5. Temperature variations of iron cylinders (on iron surface plate)

5.2 Deformation

The second most influencial factor (next to temperature) for error is deformation. Deformation can be caused by: (1) force exerted on the test piece by the measuring instrument, (2) supporting posture of the test piece, and (3) supporting posture of the measuring instrument.

5.2.1 Deformation by compression

When a force (within elastic limit) is applied on a test piece for measurement, the test piece is deformed. The deformation $\Delta\ell$ is given as follows by Hooke's law (see Figure 6).

$$\Delta\ell = \frac{PL}{EA} \text{ mm}$$

E : Young's modulus Kg/mm² (2×10^4 Kg/mm² for steel)

A : Cross sectional area mm²

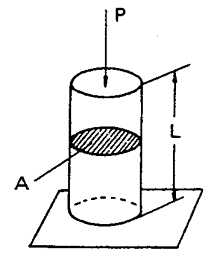


Figure 6. Deformation by compression

P : Measuring force kgf

L : Length of test piece mm

Example

(1) When a steel gauge block ($A = 9 \times 35 = 315$mm² and L = 1000mm) is measured with measuring force P = 1 kgf, deformation is given as follows.

$$\Delta\ell = \frac{1 \times 1000}{2 \times 10^4 \times 315} = 0.159 \times 10^{-3} \fallingdotseq 0.16\mu m$$

(2) When a long rod such as the standard bar of a micrometer is placed in the vertical posture, it is deformed by its own weight, that is,

$$\Delta\ell = \frac{L^2 \delta}{2E}$$

$\Delta\ell$: Deformation by its own weight mm
L : Length of test piece mm
δ : Density of test piece Kg/mm³
E : Young's modulus

Then, deformation is

$$\Delta\ell = \frac{L^2 \times 7.9 \times 10^{-6}}{2 \times 2 \times 10^4} = L^2 \times 2 \times 10^{-10}$$

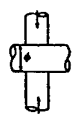
If L = 1000mm, $\Delta\ell = 0.2\mu m$. This type of deformation can be neglected if the test piece is short; however, note must be taken when it is long.

5.2.2 Contact of bent surface

When the measuring face has a radius of curvature and the measuring force applied is within the elastic limit, elastic deformation occurs on the contact surface. If the force is more than the limit, plastic deformation occurs. Because of the deformation, the test piece and measuring instrument make face-contact and error δ is produced which can be eluded in geometric point contact and line contact where measuring force is zero.

The error is calculated by Hertz with several assumptions. See JIS B-0271. Table 2 lists the errors in face contacts calculated according to Hertz's formulae.

Table 2. Deformations of steel when measurement force of 1 Kg is applied (example)

Condition of contact	Ball between two flat surfaces	Cylinder between two flat surfaces	Two balls	Plate between two balls
Illustration				
Radius of contact surface (mm)	Sum of deformations at contact points (μ)			
0.5	4.8	1.2	3.1	4.8
1	3.8	0.92	2.5	3.8
2	3.0	0.73	2.0	3.0
5	2.2	0.54	1.5	2.2
10	1.8	0.43	1.2	1.8
20	1.4	0.34	0.9	1.4
50	1.0	0.25	0.7	1.0
100	0.8	0.20	0.5	0.8
200	0.6	0.16	0.4	0.6

5.2.3 Deformation by supporting method

When the standard bar of a micrometer or standard scale is directly placed on a flat surface like surface plate, the unevenness of the surface produces errors which depend on where it is put. That is, if the surface has a radius of curvature, it is equivalent to be placed on the uneven surface and accuracy is aggravated. In this case, knife edges or cylinders must be used to support the bar on the symmetrical points.

(1) Airy points

For an end gage like gauge block, it must be supported so that two ends become parallel. The supporting points are given by the fomula below:

$$d = \frac{\ell}{\sqrt{N^2 - 1}}$$

For two-point support, $d \fallingdotseq 0.5774\ell$. That is, it must be supported at points 0.2113 apart from the both ends. See Figure 7(a).

(2) Bessel points

These points apply in two-point support of a rod to minimize the contract of the center line in length. Therefore, this way is most suitable for supporting a standard scale, having graduations on the center line. The Bessel points are apart from the end point by 0.2203ℓ respectively, and d' is measured as 0.5594ℓ. Fig. 7(b)

In the supporting method shown in Figure 7(c), the flexures at the center and both ends are the same. And the overall amount of flexure is at minimum. Therefore, the method is appropriate for a straight scale.

In the supporting method shown in Figure 7(d), the flexure between the supporting points is the smallest. This method is suitable to use the central part of the rod as a straight scale.

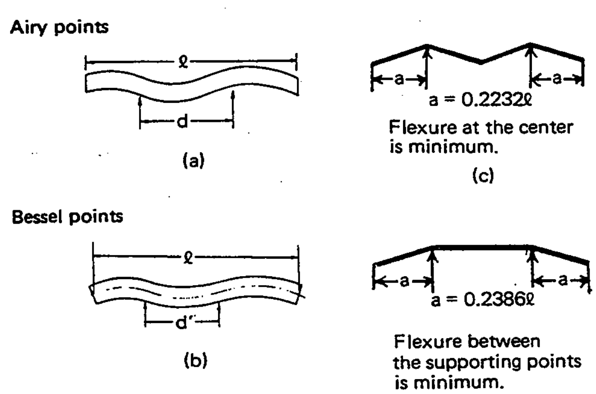


Figure 7

5.2.4. Deformation of measuring head support

The arm and column of the stand for a dial gage or electric micrometer are bent when measuring force is applied on them. See Figure 8. However, if a constant force is applied at each time, measurement error is not produced.
Error due to the flexure by measuring force is given by the following formula.

$$\delta = \frac{P\ell^2 L_2}{EI} + \frac{P}{3} \cdot \frac{\ell^3}{E'I'}$$

I, I' : Moments of inertia of cross sections of stand and arm, respectively
E, E': Young's moduli of stand and arm, respectively

The error is proportional to the square plus cubic of the arm length; therefore, stretching the arm must be avoided for the dial gage stand.

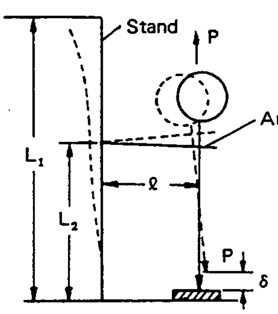


Figure 8. Bending of measuring head supporting portion

5.3 Abbe's Law

The standard scale and workpiece must be aligned on the same line of measurement. This was announced by Abbe of Carl Zeis in 1890.

For example, the law is explained on the micrometers: bench micrometer and caliper type micrometer. See Figure 9. For the ease of explanation, their measuring faces are assumed to be round shaped. The deviation angle of the measuring instrument is assumed to be θ.

(a) For the bench micrometer, error ξ_a is as follows.

$$\xi_a = L - \ell = (1 - \cos\theta)$$
$$= L\left(1 - 1 + \frac{\theta^2}{2} + \frac{\theta^4}{2^4} + \ldots\ldots\right) \doteqdot \frac{L}{2}\theta^2$$

(b) For the caliper type micrometer, error ξ_b is as follows.

$$\xi_b = \ell - L = R\tan\theta = R\left(\theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots\ldots\right)$$
$$\doteqdot R\theta$$

The error is proportional to the square of θ in (a), and is proportinal to θ in (b).

If θ, R, and L are as follows:

θ = 1' $\doteqdot$ (1/3000 radians)
R = 1' = 30mm
L = 30mm,

$$\xi_a = \frac{30}{2}(1/3000)^2 = 0.002\mu m \text{ for (a)}$$

and

$$\xi_b = 20 \times 1/3000 = 10\mu m \text{ for (b)}.$$

Thus, there is a big difference between the above two types of micrometers. In an actual measurement, error incurred by the forms of the measuring faces is produced too.

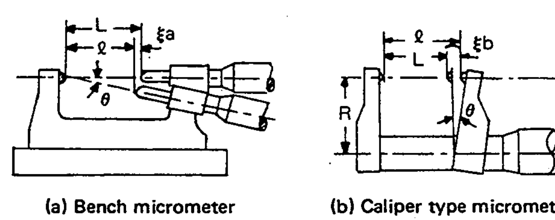


Figure 9

5.4 Parallax Error

In ordinary vernier calipers and micrometer, there is a space between two graduated faces. This gives rise to an error depending on the angle in which the inspector looks at them. See Figure 10.

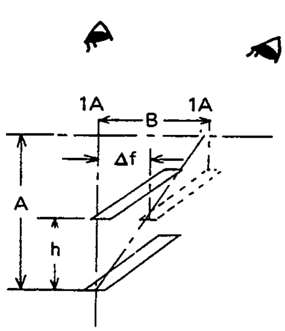
Example
In the following condition:

A = 200mm (distance of clear vision)
B = 30mm
h = 0.15mm,

parallax error is given as follows

$$\Delta f = \frac{30}{200} \times 0.15 \doteqdot 0.023\text{mm}.$$

This means when there is a space between two graduation lines on different faces, the inspector must look at them in the direction perpendicular to the faces with one eye. In order to prevent parallax error, measuring instruments with graduation lines on the same plane are manufactured. In an experiment to find the degree of parallax at the time fifty women use vernier calipers, dispersion was 0.04mm.



A : Distance between the eye and lower graduation face
B : Distance between two eye – positions
h : Distance between two graduation faces
Δf : Error caused by parallax (= B/A · h)

Figure 10

5.5 Instrumental Error

The instrumental error is obtained by subtracting the true value from the value indicated by the instrument. To perform accurate measurement, the characteristics of the measuring instrument must be checked and measured values must be corrected by taking the characteristics into consideration. Note must be taken even for comparison measurement with the standard, because the standard itself is not free from error.

6. MEASUREMENT AND RECORDING

When measurement is performed, measured values are ordinarily recorded for assuring correctness. For precision measurement, it is better that two attendants be at work, one is dedicated to measuring operation and the other is specialized in recording. In this case, notes must be taken as follows.

For the operator

a. Tell the recording personnel of measured values with clear pronunciation

b. Reassure the measured value immediately after taking data to prevent misreading

c. Assure that the recording personnel verbally repeat the correct value at each time of reading data

d. Perform measurement in the same condition every time

When a knob is to be turned clockwise, it must be turned clockwise every time in a constant speed. The same thing can be said in a case where the knob or alike must be moved downward or vice versa. The operator must stand at the same place at each time, otherwise the conditons of body heat radiation onto the measuring instrument and workpiece and floor alignment alteration due to body weight movement may somewhat affect the accuracy of measurement.

For the recording personnel

a. Be sure to record date, operator, recording personnel, name of measuring instrument, start/end time, temperatures before and after measurement, place of measurement, and weather.

b. Repeat verbally the value read by the operator and be sure that the value recorded be the same as the one repeated verbally.

c. Record values correctly and do not erase data once written. The first data later corrected must be checked with a line and the word "corrected" so that it may be known later on.

d. When graph is to be drawn, write readings first and then put them on the graph.

e. When especially accurate measurement is to be performed, take note on the details of abnormalities which occurred during measurement. In a particular case, the mental condition of the operator must be recorded.

7. ANALYSIS OF MEASUREMENT ERROR

The following may cause errors.

a. Carelessness of the operator

b. Misreading of graduation or number

c. Misrecording

d. Instrumental error inherent to the measuring instrument

e. Measuring method and natural phenomenon such as heat expansion

f. Personal error produced by operator's characteristics

The above causes of error can be removed by taking caution, experience, and correction. However, if these systematic causes of error are eliminated, the measured values still contain errors. Even if measurement is repeated in the same conditions, dispersion in measured values is produced. This type of error can not be scrutinized and assumed to be caused by many unknown factors and is called random error.

The accidental error can be reduced by paying special attention during measurement and calculating the mean of the measured values.

It is impossible to obtain the true value of a quantity by measurement. Therefore, taking the mean of the measured values is the best solution to obtain a value which is as close to the true value as possible as proposed by Gauss.

Standard deviation σ is often used to indicate the degree of correctness or accuracy of the measuring instrument and measured values as follows.

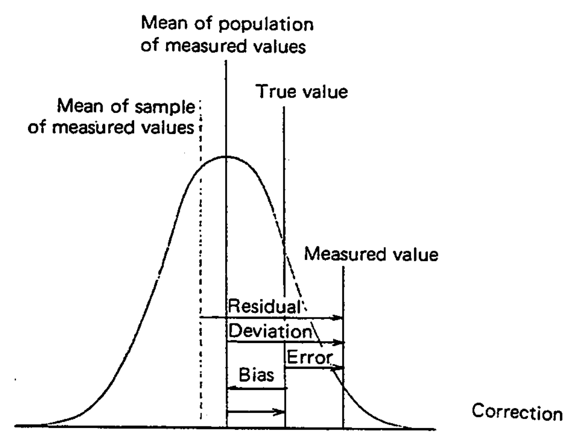


Figure 11

$$\sigma = \pm\sqrt{\frac{\Sigma V^2}{n}}$$

where, V is called residual and can be obtained by subtracting the sample mean from a measured value. See Figure 11.

When a quantity is measured n times and the measured values are assumed to be $M_1, M_2 \ldots\ldots M_n$, the mean X_0 is

$$X_0 = \frac{M_1 + M_2 + \ldots\ldots + M_n}{n} = \frac{\Sigma M}{n} \qquad (1)$$

If $V_1, V_2, \ldots\ldots V_n$ are defined as follows:

$$M_1 - X_0 = V_1$$
$$M_2 - X_0 = V_2$$
$$\vdots$$
$$M_n - X_0 = V_n,$$

the sum of the above equations is

$$\Sigma M - nX_0 = \Sigma V. \qquad (2)$$

$\Sigma V = 0$ is deduced from equations (1) and (2) and is the feature of the mean.

Example

A quantity is measured ten times and the following values are assumed to be obtained.

Times	Measured value	V	V^2
1	3.57	+0.013	0.0002
2	3.54	–0.017	0.0003
3	3.56	+0.003	0.0000
4	3.55	–0.007	0.0001
5	3.58	+0.023	0.0005
6	3.54	–0.017	0.0003
7	3.55	–0.007	0.0001
8	3.57	+0.013	0.0002
9	3.56	+0.003	0.0001
10	3.55	–0.007	0.0001
	ΣM=35.57	ΣV= 0	ΣV^2=0.0019

Therefore

$$\sigma = \pm\sqrt{\frac{0.0019}{10}} = \pm\, 0.014$$

8. INTERFERENCE OF LIGHT WAVES

The light is electromagnetic wave in a certain range of wave length as proved by physics.

The interference of light waves occurs when two light waves emitted from two different sources meet each other and their amplitudes are algebraically added. Figure 12 illustrates how the two light waves with the same wave length interfere with each other; and the broken/chain lines and full line represent the light waves emitted from the two sources and resultant wave, respectively.

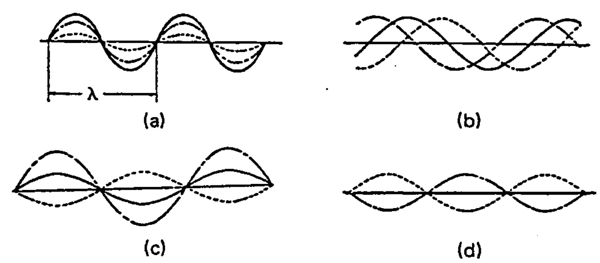


Figure 12

(a) The two waves are the same in phase and the resultant wave becomes brighter.

(b) One light wave (broken line) leads in phase the other light wave (chain line) by three-eighths of the wave length. In this case, the resultant light wave is at the mid points between the two waves in amplitude and phase.

(c) The phases of the two light waves differ from each other by half wave length and the resultant wave is biased on the side of the original wave with greater amplitude.

(d) Phase difference is the same as (c) above and the amplitudes are the same. The resultant amplitude is zero and it is dark.

When two sheets (P_1 and P_2) of polished glass with high flatness and half plated are placed being slanted with each other by a slight angle of α and a monochromatic light (blue-green light emitted from a mercury lamp) is radiated to the faces, interference fringes appear. The fringes are in parallel with the edge line of the wedge formed by two glass surfaces and are in equal intervals which are proportional to the wave length of the monochromatic light. This phenomenon is called interference of equal thickness. See Figure 13.

The monochromatic light (wave length λ) passes through the glass sheet P_1 in the direction ab and then is reflected on the surface of the glass sheet P_2 in the direction ce and also passes through it being refracted in the direction cd. The light ce is further reflected on P_1 and proceeds in the path e → f → g. See Figure 14.

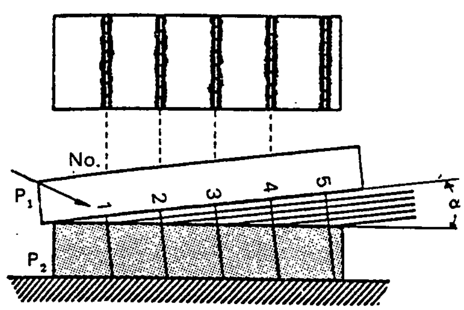


Figure 13

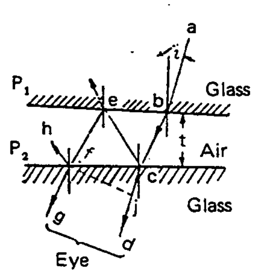


Figure 14

The surfaces of P_1 and P_2 are slightly slanted with each other and so cd and fg are not exactly in parallel. However, if a suitable lens is used to converge them, interference phenomenon caused by the difference between their phases can be observed. If the thickness of the air space is t_1 and the angle of incidence is i, the path difference δ between cd and fg is

$$\delta = 2t \cos i.$$

If the path difference δ is

$$\delta = (2n+1)\,\lambda/2 \qquad \text{(n: positive integer)},$$

that is,

$$2t \cos i = (2n+1)\,\lambda/2,$$

the two light waves reduce their intensities with each other and a dark line appears.

On the contrary, if the thickness t satisfies

$$\delta = 2t \cos i = 2n\,\lambda/2,$$

the two light waves enhance their intensities with each other and a bright line appears.

As shown by the above equations, the appearance of the interference fringes depends on the angle of incidence i. Therefore, parallel light waves (that is, the angles of incidence are the same) must be used to obtain a clear picture of interference fringes.

If i = 0, that is, the light waves are perpendicularly incident to the glass surface P_1, a bright line appears in the location

$$t = 2n\,\lambda/4 \text{ (multiplied by an even number).}$$

A dark line appears in the location

$$t = (2n+1)\lambda/4 \text{ (multiplied by an odd number).}$$

If the glass surfaces are flat completely and perfectly, the locus along the air gap of an equal space is a straight line in parallel with the intersection line of the two surfaces. Therefore, interference fringes are in parallel with each other. The difference of air space between adjacent bright (or dark) line locations is

$$(2n + 1)\,\lambda/4 - (2n - 1)\,\lambda/4 = \lambda/2.$$

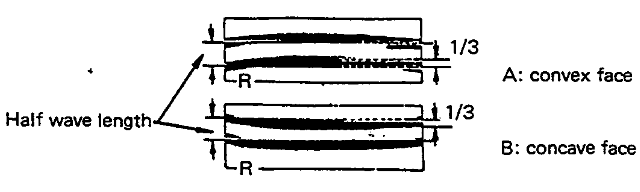
This means that the next fringe appears at every difference of air space which is equal to $\lambda/2$; and the smaller is the angle α between the two glass surfaces slant with each other, the wider is the intervals between fringes; and the longer is the wave length of the monochromatic light, also the wider are the intervals. The interval S between adjacent bright (or dark) lines is

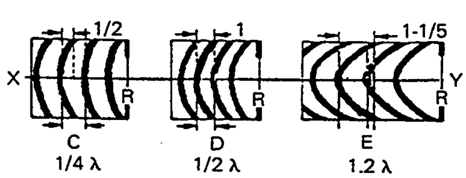
$$S = \frac{\frac{\lambda}{2}}{\tan\alpha} = \frac{\lambda}{2\tan\alpha} \fallingdotseq \frac{\lambda}{2\alpha}$$

If one of the glass surface is not flat enough, the pattern of the fringes is like contour lines of its topographic map. That is, contour lines are closer to each other in steep slopes and extended in gentle slopes.

If white light is used, several fringes with different colors appear only at the location t = 0 where path differences are zero and do not appear at distant locations, because interferences of different wave lengths occur at the same time.

The principle of interference of equal thickness is widely used for measuring: the size of gauge block; flatness by means of optical flat; parallelism by means of optical parallel; surface roughness by means of interference microscope.





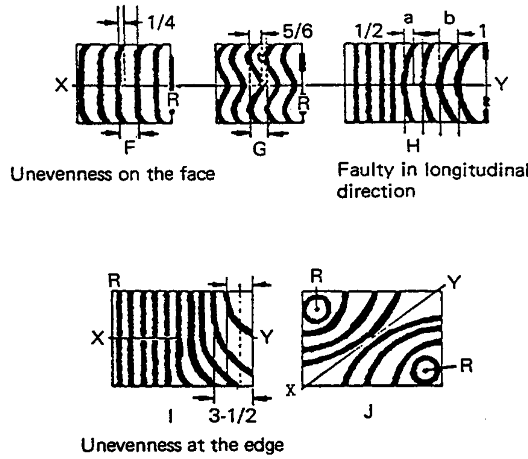


Fig. 15 Interference

9. CORRECT USE OF MEASURING INSTRUMENT

When a measurement is to be made in a factory, a suitable instrument must be selected in accordance with the accuracy specified for the workpiece. In some cases, a special measuring instrument must be used depending on the material and form of the workpiece.

As described in section 4.1 Selection of measuring instruments, the following are the fundamentals of accurate measurement:

- Select a suitable measuring instrument in accordance with the accuracy specified for the workpiece,
- Determine the method of measurement,
- Take environmental conditions causing errors into consideration,
- Take data several times carefully, and
- Analyze resultant data theoretically.

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