TEXTBOOK NO. 7022④

# Fundamentals of Optics

## TEXTBOOK

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CONTENTS

## 1. PREFACE

Scientific studies on optics today focus mainly on developing laser and holography technology. However, instruments based on optical lens systems are still the principal applications in the industrial field. The purpose of this textbook is to introduce the basic principles of geometrical and physical optics as a means to a better understanding of optical instruments.

## 2. BRIEF HISTORICAL INTRODUCTION

The first known study of geometrical optics is credited to the Greek philosopher Euclid around 300 B.C. In his major work "Elements", the reflection of light was discussed and these ideas were later developed by Archimedes and others. Around 1255, an Italian by the name of Salvino degli Armati is said to have invented the spectacle lens and at the turn of the 13th century, production of these lenses had already begun in Italy. The invention of the telescope is generally credited to a Dutch lens grinder named Zacharias Janssen, in 1604. In the same century, Kepler, Fermat, and Descartes developed fundamental lens theory concerning the properties of reflection and refraction. As the need for telescopes capable of meeting the rapidly developing science of astronomy grew, the problem of aberrations and flaws in the lenses of the larger telescopes prompted the production of the reflecting telescope, to which the works of Newton played an important role. From the late 18th century into the 19th century, Gauss and Von Seidel published the results of their research into lens design and aberration, which, with the development of new types of optical glass, opened the way for modern optical technology.
Recently, research is being actively pursued into developing lenses almost free from aberrations (used for IC production) and zoom lenses, using the latest technological advances in antireflection coatings and computer-aided lens design.
Since the time the maser theory was proposed by C.H. Townes in 1952, technological advancement in this field has progressed rapidly, resulting in products such as laser interferometric measuring instruments and various holographic-application devices.

## 3. GEOMETRICAL OPTICS

### 3.1 Reflection and Refraction

#### 3.1.1 Rectilinear propagation of light
The basic concept of geometrical optics is a simple, everyday notion - light travels in a straight line unless it is reflected or refracted. A substance that allows light to pass is called the *medium*, and light traveling in a homogeneous medium will propagate in a straight line. This is known as *the law of rectilinear propagation of light*. The straight line along which light passes is called a *ray* and a collection of rays is called a *bundle or pencil of rays*. When collective rays diverge from a point source, they are called a *divergent rays*; when they gather at a point, they are called a *convergent rays* and when they propagate along parallel paths, they are called *parallel rays*.

#### 3.1.2 Law of reflection
As light meets the boundary of two, different mediums, part of the light is reflected. In **Fig.1**, point B is called the *point of incidence* with line A-B representing the *incident* ray and B-C the *reflected* ray. The angles $\alpha$ and $\alpha'$, which the incident and reflected rays make with B-N, the normal or perpendicular to the reflecting surface at the point of incidence, A-B-N is called the *angle of incidence* and N-B-C is called the *angle of reflection.*
The plane that includes the two rays is called the *plane of incidence*.

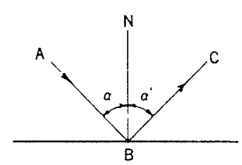



**Fig.1** Reflection of light

The laws of reflection can be summarized as follows:
(1) The incident and the reflected rays are on opposite sides of the normal at the point of incidence and all three lie in the same plane.
(2) The angle of incidence is equal to the angle of reflection.

Mirrors and sheets of glass are highly polished surfaces, and so each ray in a pencil is reflected in the same way; this is called *regular reflection.* The surfaces of most objects, however, have tiny irregularities, and when a parallel beam of light falls on such a surface, the individual rays strike it at different angles of incidence.

The rays are, therefore, reflected in different directions from the surface; this phenomenon is called *diffused reflection* and the light reflected is called *scattered light.*

### 3.1.3 Law of refraction

As light meets the boundary of two different mediums as shown in **Fig. 2**, part of the light is reflected and the remainder crosses the boundary into medium 2. At this point the light changes direction; this is known as *refraction.*

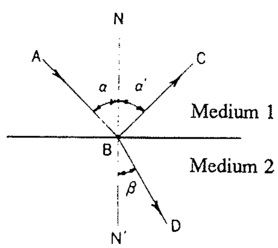



**Fig.2** Refraction of light

In **Fig. 2**, the angle of incidence α is the angle between the incident ray and the normal to the refracting surface at the point of incidence, (A-B-N). The *angle of refraction* β is the angle between the refracted ray and the normal, (N′-B-D). The laws of refraction can be stated as follows:

(1) The incident and the refracted rays are on opposite sides of the normal at the point of incidence and all three lie in the same plane.

(2) The ratio of the sine of the angle (α in this example) of incidence to the sine of the angle (β) of refraction is a constant for a given pair of media (also known as Snell's laws).

The value of the constant for a ray passing from one medium to another is called the *refractive index* of the second medium with respect to the first and is denoted by the symbol n. This is also known as the relative refractive index.

$$\frac{\sin \alpha}{\sin \beta} = {}_1n_2$$

If, however, the first medium is air, it is usual to speak of n simply as the refractive index of the second medium but, strictly speaking, the *absolute* refractive index of a medium is the value of n when the first medium is a vacuum. When the absolute refractive indexes of medium 1 and medium 2 are given by $n_1$ and $n_2$ respectively, the relationship between the relative index and absolute index is expressed by the following formula:

$${}_1n_2 = \frac{n_2}{n_1}$$

The refractive index (absolute) of air is close to 1, and, for most practical purposes, it is usual to use 1 as the approximate refractive index of air.

### 3.1.4 Principle of Reversibility of Light

The principle of reversibility of light states that for any optical system the paths of light rays are reversible.

### 3.1.5 Total internal reflection

When light passes from one medium to a more optically dense medium, there will always be both reflection and refraction for all angles. This is not the case when light passes from one medium to a less optically dense medium. In this case the angle of refraction is greater than the angle of reflection. As the angle of incidence increases, the angle of refraction also increases, and at the same time the intensity of the reflected rays gets stronger and that of the refracted rays weaker. Finally, at a certain angle called the *critical angle of incidence* ø, the angle of refraction becomes 90°. Since it is impossible to have an angle of refraction greater than 90° it follows that for angles greater than the critical angle ø all incident light is reflected at the boundary; this is what we describe as *total internal reflection.*

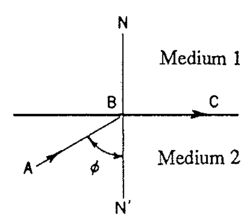



**Fig.3** Total refraction

The critical angle of refraction ø is given by the following formula:

$$\frac{\sin \phi}{\sin 90^\circ} = {}_2n_1$$

Therefore,

$$\sin \phi = {}_2n_1 = \frac{1}{{}_1n_2}$$

Worked example:

Prove that ${}_2n_1 = \frac{1}{{}_1n_2}$

Proof:
Let the incident angle equal a and the angle of refraction equal b, and assume that light propagates from medium 1 to medium 2. As shown in the previous section,

$$\frac{\sin\alpha}{\sin\beta} = {}_1n_2$$

From the principle of reversibility of light, we obtain

$$\frac{\sin\beta}{\sin\alpha} = {}_2n_1$$

Therefore,

$${}_2n_1 = \frac{1}{{}_1n_2}$$

## 3.2 Mirrors

### 3.2.1 Plane mirrors

The image formed by plane mirror reflection is symmetrical and face-to-face with the original object.

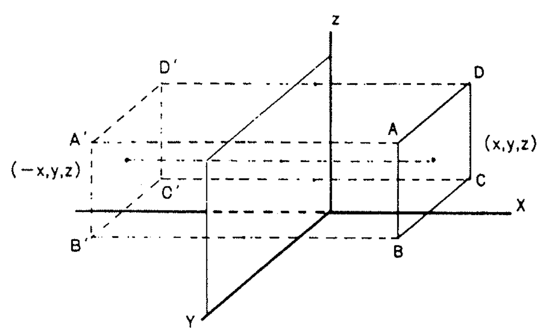



**Fig.4** Symmetrical image formation by a plane mirror

When a rectangular coordinate system is reflected by a mirror on the Z-Y plane, the relationship between the object point (x, y, z) and the image point (x′, y′, z′) is given as x′=–x, y′=y and z′=z. This means that the dimensions of the image and object are same, and that the image and object are the same distances from the mirror surface. The image is a *lateral inversion* of the object.

As shown in **Fig. 5**, the diverging rays that propagate from point source S are reflected by the plane mirror and continue in such a way that they appears as if the rays were emitted from point source S′ at symmetrical point S on the mirror surface. Point S′ is called the *virtual image.*
When rays that would converge at a point S are reflected by a plane mirror, the reflected rays will converge at point S′ (**Fig.6**). Point S′ is called the *real image.*

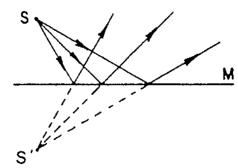



**Fig. 5** A virtual image formed by plane mirror reflection

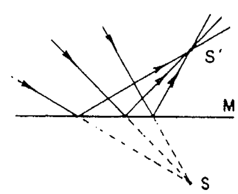



**Fig. 6** A real image formed by plane mirror reflection

### 3.2.2 Spherical mirrors

In this section, we shall look at the way in which images are formed by curved mirrors. The kind we use are generally made by silvering a piece of glass which would form part of a hollow sphere. Silvering the glass on the outside gives a concave or converging mirror, while silvering on the inside gives a convex or diverging mirror.
The center of the mirror is called the vertex and the line that joins the mirror center to the center of curvature (the center of the sphere of which the mirror forms a part) is called the *principal axis* . The angle formed by the principal axis and the line connecting the spherical center to the mirror rim is called the *mirror angle.* In addition, the angle formed by the principal axis and a ray of light is called the *ray angle.* The following formula shows the conditions concerning image formation of a spherical mirror. **(Fig. 7)**

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r} = \frac{1}{f}$$

Here, a, b, r, and f are positive values when measured at the front side of the mirror face and negative values when measured at the rear.

$a > 0$ real object
$a < 0$ virtual object
$b > 0$ real image
$b < 0$ virtual image
$r > 0$ concave mirror
$r < 0$ convex mirror

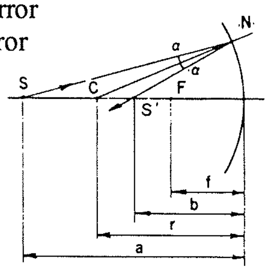



**Fig. 7** Image formation by a spherical mirror

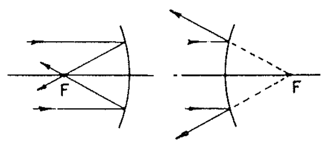



**Fig. 8** The focal point of a spherical mirror

The rules for constructing images formed by spherical mirrors can be summed-up as follows:
(1) Rays passing through the center of curvature are reflected back along their own paths.
(2) Rays parallel to the axis are reflected through the focal point.
(3) Rays passing through the focal point are reflected parallel to the principal axis.

These propositions only apply to mirrors that are small in size or aperture compared with their radius of curvature, but are not applicable when it is large. When the mirror's subtended angle is large, the reflected rays do not necessarily converge into a focal point, as shown in **Fig.9** but instead form a surface called a *caustic*. These three propositions, however, are absolutely true with parabolic mirrors even when the mirror angle is large; all the reflected rays converge into a focal point.
The magnification of a spherical mirror is given by the following formula:

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{B}{A} = -\frac{f}{a-f}$$

$$= -\frac{r}{2a-r}$$

Here, m is a positive value for erect images, and a negative value for inverted images.

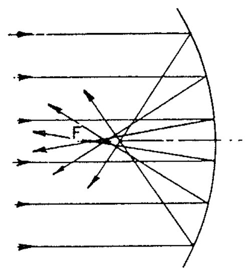



**Fig. 9** Image formation when the mirror angle is large

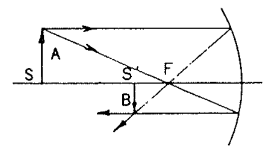



**Fig. 10** Magnification of a spherical mirror

Worked example:
An object 1 cm high is placed on the principal axis in front of a convex mirror whose radius of curvature is 10 cm and at a distance of 20 cm from the mirror center. Explain the image that would be formed.

Answer:
Take the identity previously given above

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

Therefore,

$$b = \frac{ar}{2a-r}$$

Take the identities also given above

$$m = -\frac{f}{a-f}$$

$$B = mA$$

By substituting the given values for a, r, f (=r/2) and A into these formulae, the values of b, m and B are obtained as follows:

$$b = -4 \text{ cm}, m = 0.2, B = 0.2 \text{ cm}$$

This means that an erect virtual image 0.2 cm high is formed on the principal axis 4 cm behind the mirror face.

### 3.3 Lenses

#### 3.3.1 Paraxial range

Incident rays passing through a lens follow the laws of refraction, and to cope with this situation we must use trigonometric functions. Approximating formulae for trigonometric functions are expressed by the following series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\cdots$$

$$\cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots\cdots$$

Here, the range for which the conditions $\sin x = x$, $\cos x = 1$, $\tan x = x$ are satisfied (the 2nd term of the series being small enough to ignore ) is called the *paraxial range.* Rays within the paraxial range are called *paraxial rays* and are supposed to form an ideal image when the light used is *monochromatic* (of a single wavelength).

### 3.3.2 Ray tracing and positive/negative sign convention

Assume an incident ray traveling in the direction A-S crosses the spherical boundary of two different media, each refractive index, n and n′, changes the course taken by the line A-S′ after refraction. The incident ray A-S is defined by angle u, formed by the ray and the axis, and the length BS = ℓ is given by the distance along the axis from the boundary to the intersection point of line A-S and the axis (**Fig.11**). Here, distance ℓ is a positive value when BS is measured from the boundary in the direction of the incident ray, and a negative value when measured in the opposite direction. The height h, the length of the perpendicular line measured from incident point to the axis, takes a positive value when it is on the upper side of the axis, and a negative value when on the lower side. The positive/negative sign of angle u is determined by the signs of h and ℓ. The radius of curvature r of the sphere takes a positive value when the incident ray crosses the convex side of the curvature first, and a negative value when it crosses the concave side first. The incident angle i and the angle ø, which are formed by the line AC connecting the point of incidence and the center of the sphere and the optical axis, are determined by h and r. The positive/negative signs of the angles u and ø are determined in such a way that the angles are positive when they make an acute angle clockwise with respect to the optical axis, and a negative value when opposite. The sign of angle i is determined similarly, but the base line taken is the ray line instead of the optical axis.

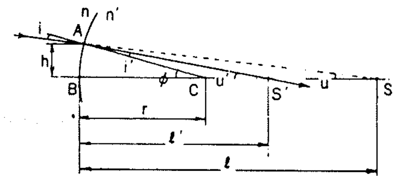



**Fig.11** Lens image formation

Ray tracing (calculation of the paths taken by rays of light through an optical system) is carried out based on the following formula.

1) Paraxial ray tracing formula

$$\frac{1}{\ell'_v} = \frac{n_v}{n_{v+1}\ell_v} + \frac{1}{r_v}\,\frac{n_{v+1} - n_v}{n_{v+1}}$$

$$\ell_{v+1} = \ell'_v - d_v$$

In these formulae, the value of $\ell_{v+1}$ is obtained by using a known value of $\ell_v$

2) Marginal ray tracing formula

$$\frac{\sin i_v}{\sin u_v} = \frac{\ell_v - r_v}{r_v}$$

$$n_{v+1} \sin i'_v = n_v \sin i_v$$

$$u'_v = i_v - i'_v + u_v$$

$$\frac{\sin i'_v}{\sin u'_v} = \frac{\ell'_v - r_v}{r_v}$$

$$u_{v+1} = u'_v$$

$$\ell_{v+1} = \ell'_v - d_v$$

$$h_v = r_v \sin(u_v + i_v)$$

In these formulae, the values of $\ell_{v+1}$ and $u_{v+1}$ can be obtained by using known values for $\ell_v$ and $u_v$. When the vth refracting surface is a plane, the following formulae are used.

$$i_v = u_v$$

$$n_{v+1} \sin i'_v = n_v \sin i_v$$

$$u'_v = i'_v$$

$$\frac{\ell'_v}{\ell_v} = \frac{\tan u_v}{\tan u'_v}$$

$$u_{v+1} = u'_v$$

$$\ell_{v+1} = \ell'_v - d'_v$$

$$h_v = \ell'_v \tan u'_v$$

From these formulae, the values of $\ell_{v+1}$ and $u_{v+1}$ can be obtained.

In a lens system, the point of image formation of rays emitted from an object can be traced by applying these formulae to each refracting surface sequentially.

### 3.3.3 Thin lenses

It is often convenient to consider a lens by ignoring its thickness for study purposes; lenses treated in such a way are called *thin lens*. At the initial stage of lens design work, this concept is frequently employed as it can simplify calculations to a great extent. The formula to give the conditions of thin lens image formation is as follows.

$$\frac{1}{b} - \frac{1}{a} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{1}{f}$$

where
n = refractive index of lens
ri = radius of curvature of lens
f = focal length of lens

The magnification, m, is given by the following formula.

$$m = \frac{b}{a}$$

When $m > 0$ an erect image is formed
When $m < 0$ an inverted image is formed

Here, we consider the case of a combination of thin lenses. If the distance between the thin lenses can be ignored, the focal length of the combined lens system F is given by the following formula.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

This system is considered to be identical to a thin lens having a focal length F.

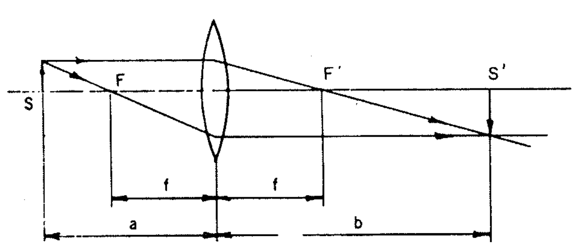



**Fig.12** Thin lenses image formation

### 3.3.4 Thick lenses

In the case of a thick lens, given HS = a, H´S´ = b (**Fig.13**), the following formula is used to obtain the conditions of image formation.

$$\frac{1}{b} - \frac{1}{a} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2 d}{n r_1 r_2}$$

$$= \frac{1}{f}$$

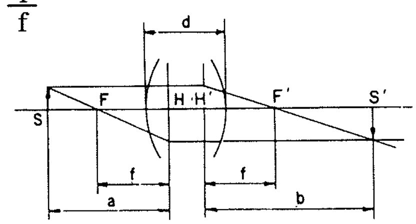



**Fig. 13** Thick lenses image formation

When the incoming incident ray reaching point H and the emergent ray leaving point H´ (**Fig.14**) satisfy the relationship nu = n´u´, then these points are called *principal points*.

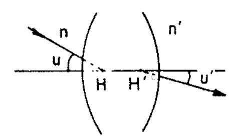



**Fig.14** The principal points of a lens

When the incoming incident ray reaching point K and the emergent ray leaving point K´ (**Fig.15**) satisfy the relationship u = u´, then these points are called *nodal points*. Consequently, when the incident ray is directed to the nodal point, the ray propagates parallel to the incident ray after passing through the lens. When n = n´, (when the refractive index of the object field and that of the image field) are identical, points H and K are the same, and so with points H´ and K´.

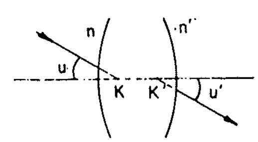



**Fig.15** The nodal points of a lens

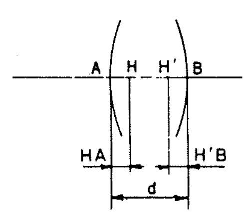



**Fig.16** The principal points of a lens

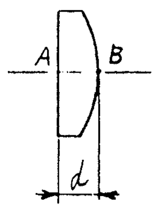
For a single lens system, the distances from the principal points to the lens surface are given by the following formula (**Fig.16**).

$$HA = \frac{r_1 d}{n(r_2 - r_1) + (n-1)d}$$

$$H'A = \frac{r_2 d}{n(r_2 - r_1) + (n-1)d}$$

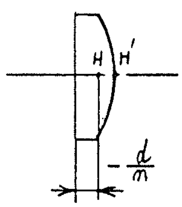
where
n = refractive index of the lens
ri = radius of curvature of the lens

Worked example:
Obtain the principal point positions and the focal length of a plano-convex lens.




Answer:

$$HA = -\frac{d}{n}$$
$$H'B = 0$$
$$f = -\frac{r_2}{n-1}$$




### 3.3.5 Antireflection coatings

When light enters a lens, about 5% of the light is lost due to reflection at the lens surface. For a small optical system this loss is usually insignificant, but as the number of lenses in a system increases, the loss becomes substantial. For example, the transmittivity (transmission factor) of a 10-lens system is reduced to about 36%. In order to minimize this loss, a coating satisfying the following conditions is required. Let the refractive index of glass be ng and the wavelength of light be $\lambda$. There, the conditions to eliminate reflection are attained by applying a transparent coating, whose refractive index $n = \sqrt{ng}$, with a thickness d, where d is defined by the formula $nd = \frac{1}{4}\lambda$, on the reflecting surface. This is called an *antireflection coating.*

1) Single-layer antireflection film
When applying a single-layer antireflection film, reflection decreases if $n < ng$ (the refractive index of film is smaller than that of glass), and increases if $n > ng$. No reflection takes place under the following conditions.

$$n = \sqrt{ng}$$
$$nd = \frac{1}{4}\lambda$$

When ng = 1.51633 (glass spec. BK-7), then n can be approximated as $n \fallingdotseq 1.23$. However, there is no material that has refractive index as low as this. Therefore, as a matter of practical application, we use magnesium fluoride ($MgF_2$, n = 1.391) or cryolite ($3NaF \cdot A\ell F_3$, n = 1.339) as the closest materials available, and arrange the film thickness d to satisfy the condition nd = 0.137 μm, so that the minimum reflection occurs for yellow-green light whose wavelength is around 0.55 μm. Here, nd is called the *optical thickness..* When nd $\fallingdotseq$ 0.137 μm, the surface is tinged a reddish-purple color; this is called a magenta coating. When nd $\fallingdotseq$ 0.115 μm, it is called amber coating as the surface is an amber-tinged and when nd $\fallingdotseq$ 0.147 , a blue coating. Generally, the magenta coating is most frequently used.

2) Multi-layer antireflection film
Several layers or coatings (a succession of thin films), each having a thickness about the same as the wavelength of light, can be applied to a lens. This is called a *multilayer film.* A two-layer film is used to minimize the reflection at two different wavelengths; three-layer film for three different wavelengths, etc. As three-layer films can reduce the reflectivity to 0.5% or lower in the visible region, they are used for special-purpose lenses.

### 3.4 Lens Aberration

There are seven types of lens aberration, with the final two on the list below being wavelength dependant.

1) Spherical aberration
When parallel rays or rays emitted from a point source on the principal axis form an image, a phenomenon in which the rays fail to converge at one point and form a blurred circle is called *spherical aberration.*

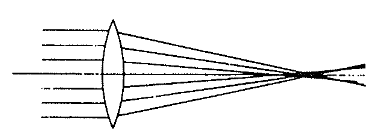


**Fig.17** Spherical aberration of parallel rays

2) Coma aberration
When rays are emitted from a light source not on the principal axis, a phenomenon in which the rays do not converge at one point and form a comet-shape image is called *coma aberration.*

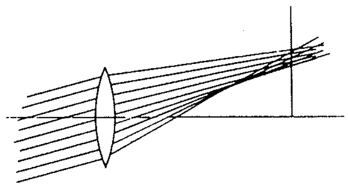


**Fig. 18** Coma aberration

3) Astigmatism
When rays are emitted from a light source not on the principal axis, a phenomenon in which the image formed by the vertical section elements of the lens and the image formed by the horizontal section elements do not converge at one point is called *astigmatism.*

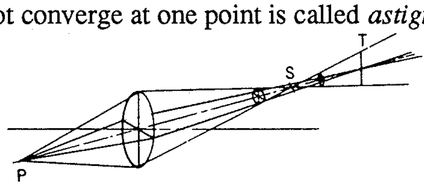



**Fig.19** Astigmatism

4) Curvature of field
Even when the rays emitted from a point source converged at one point, there are cases where the image of an object on a flat plane at right angles to the optical axis is formed on a curved plane, thus the image is distorted. This makes it impossible to get a clear image in the center region and the extremities at the same time. This phenomenon is called *curvature of field* aberration.

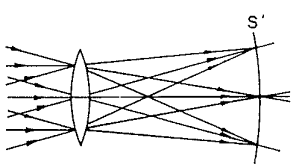



**Fig.20** Curvature of field

5) Distortion
A phenomenon in which straight lines on a plane at right angles to the optical axis but not crossing the optical axis form curved lines as shown in **Fig.21.** This is called *distortion.*

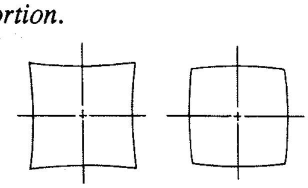


**Fig.21** Distortion

6) Chromatic aberration on axis
Even if there is no spherical aberration for monochromatic light, rays of different wavelengths may not converge at one point, causing color fringes. This phenomenon is called *chromatic aberration on axis.*

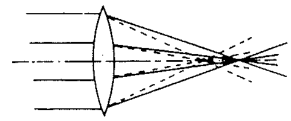


**Fig. 22** Chromatic aberration on axis

7) Chromatic aberration of magnification
When the magnification of the image varies with the wavelengths of light, it is called *chromatic aberration of magnification.*

Both types of aberration in 6) and 7) above are sometimes called simply chromatic aberration without distinguishing between the two. Of the above types of aberrations, those lenses in which spherical aberration and coma aberration have been eliminated are called *aplanat*; those in which astigmatism and curvature of field have been eliminated are called *anastigmat.* A combination of lenses in which chromatic aberration is eliminated in respect to the two spectral lines C and F is called *achromat*, and one for the C-line, d-line (or D-line) and F-line is called *apochromat.* For the objective lens of a telescope, aplanat-achromat is good enough, but photographic lenses are required to be anastigmat in addition to the other anti-aberration properties.

### 3.5 Prisms

Optical instruments make use of mirrors and prisms as design devices for changing the optical axis and image orientation. Among the prisms most commonly used are the right-angle prism, the Porro prism, the Dach prism (roof prism) and the pentagonal prism.

1) Right-angle prisms
The right-angle prism is a type of prism that has one right angle, and is the type most frequently used. It turns a beam of light through a right angle or reverses the direction of light, according to the position of the prism.

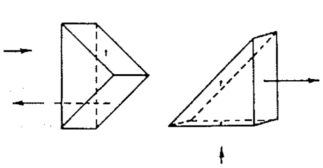


**Fig.23** Right-angle prism

2) Porro prism
The Porro prism is also called an erecting prism, and is constructed as a combination of right-angle prisms. There are two types of Porro prism, described below.

a) Type-1 Porro prism
A representative design of a Type-1 Porro prism is combination of two right-angle prisms as shown in **Fig. 24.** The image is inverted both horizontally and vertically. This type is commonly used for binoculars.

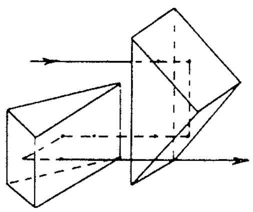


**Fig.24** Type-1 Porro prism

b) Type-2 Porro prism
Type-2 porro prism is combination of three right-angle prisms, and the image is inverted both horizontally and vertically. This is commonly used in the telescopes of surveying instruments.

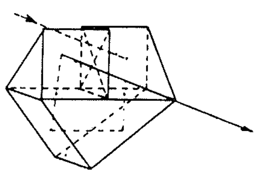


**Fig.25** Type-2 Porro prism

3) Dach prism
The Dach prism is also called a roof prism. It is shaped like a roof, having two sides angled at 90° degrees. The image is inverted both horizontally and vertically. Using a Dach prism has the advantage of inverting the image without shifting the optical axis sideways.

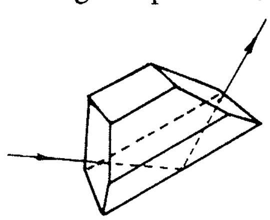


**Fig.26** Dach prism

4) The Pentagonal prism
This is a prism with a pentagonal shape, where the incident ray and the emergent ray always make a certain, constant angle regardless of the angle of incidence. A combination of a pentagonal and a Dach prism is called a *Dach-penta prism,* which is used in single-lens reflex camera.

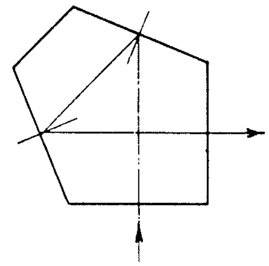


**Fig.27** Pentagonal prism

### 3.6 Dispersion of Light

A beam of white light emerging from slit and passing through a prism is refracted by the prism and separated into its component colors. This phenomenon is called the *dispersion of light.* The arrangement of monochromatic band series obtained from light dispersed by a prism or other means, is called a *spectrum.* **Table 1** shows the relationship between the spectral lines and the wavelength of the light from different sources.

**Table 1** Spectral line and wavelength

| Spectral line (Color) | C (red) | d (orange) | e (green) | F (blue) | g (indigo) | h (violet) |
|---|---|---|---|---|---|---|
| Light source | H | He | Hg | H | Hg | Hg |
| Wavelength (μm) | 656.3 | 587.6 | 546.1 | 486.1 | 435.8 | 404.7 |

# 4. PHYSICAL OPTICS

## 4.1 Interference of Light

Light, radio waves and X-rays are all electromagnetic waves, but light is differentiated from the others by its wavelengths, which are shorter than those of radio waves but longer than X-rays. When light emitted from a single point source is divided and then directed to a point so that the light waves are superimposed, the amplitude of each wave is added or canceled out, depending on the difference in their phases. This effect is known as *interference*, and the result of interference is a series of light and dark bands called *interference fringes*. Interference of light can also take place if beams of light emitted from two or more point sources have the same wavelength with definite phase relationships. In general, however, light emitted from different sources will not form interference fringes because the frequency of each light source is quite random and constantly changing, which results in waves of different and constantly varying phase. This type of light is called *incoherent* light and the most common sources are tungsten filament lamps, sunlight, etc. On the other hand, light of the same wavelength that can cause interference is called *coherent light.*

### 4.1.1 Equal thickness interference

We now consider the interference that takes place when two sheets of plane-parallel glass are positioned as shown in **Fig. 28**. The glass is semi-transparent, mirror finished with silver evaporation coatings on the inward facing surfaces. A light wave traveling in air changes phase by $\pi$ when it is reflected from the boundary of the denser medium. However, no phase change takes place when light traveling in a dense medium is reflected from the boundary of a less dense medium. We first make observations of rays $R_1$ and $R_2$.

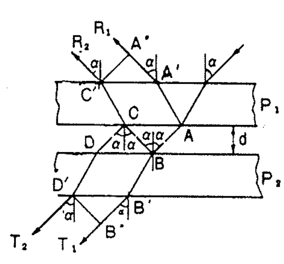



**Fig. 28** Equal thickness interference

We can obtain the optical path difference $\Delta R$ between $R_1$ and $R_2$ as follows:

$$\overline{AA''} = \overline{AA'} + \overline{A'A''}$$
$$\overline{AC'} = \overline{AB} + \overline{BC} + \overline{CC'}$$

Since $\overline{A\text{-}A'} = \overline{C\text{-}C'}$, and the $\pi$ phase change takes place at reflection point B, the optical path difference $\Delta R$ is obtained as follows:

$$\Delta R = (\overline{AB} + \overline{BC} + \frac{\lambda}{2}) - \overline{A'A''}$$
$$= 2d\cos\alpha + \frac{\lambda}{2}$$

When $\Delta R$ is an even multiple of $\lambda/2$, the amplitudes of $R_1$ and $R_2$ are added, resulting in reinforcement; when $\Delta R$ is an odd multiple of $\lambda/2$, then they cancel out each other. As a consequence, the conditions for the maximum and minimum amplitudes are:

Maximum (bright) : $2d\cos\alpha = (2n+1)\frac{\lambda}{2}$
Minimum (dark) : $2d\cos\alpha = 2n \cdot \frac{\lambda}{2}$

where n=0,1, 2, ......

The solution, in the case of interference between $T_1$ and $T_2$, is obtained in the same way.

$$\overline{BB''} = \overline{BB'} + \overline{B'B''}$$
$$\overline{BD'} = \overline{BC} + \overline{CD} + \overline{DD'}$$

Since $\overline{B\text{-}B'} = \overline{D\text{-}D'}$, and the $\pi$ phase change takes place at the two reflecting points B and C, the optical path difference $\Delta T$ is obtained as follows:

$$\Delta T = (\overline{BC} + \overline{CD} + \frac{\lambda}{2} + \frac{\lambda}{2}) - \overline{B'B''}$$
$$= 2d\cos\alpha + \lambda$$

Therefore, the conditions for maximum and minimum amplitudes are opposite to those in the case of $R_1$ and $R_2$, as follows:

Maximum (bright) : $2d\cos\alpha = 2n \cdot \frac{\lambda}{2}$
Minimum (dark) : $2d\cos\alpha = (2n+1)\frac{\lambda}{2}$
where n=0,1, 2, ......

In the case of interference between $R_1$ and $R_2$ using perpendicular light incident upon the glass plane (i.e. $\alpha = 0$), the conditions for interference are given as follows.

$$d = (2n+1)\frac{\lambda}{4} \text{(bright)}$$
$$d = 2n\frac{\lambda}{4} \text{ (dark)}$$

The relationship between the distance of the two glass sheets and the type of interference is expressed by the following formula.

$$(2n+1)\frac{\lambda}{4}-(2n-1)\frac{\lambda}{4}=\frac{\lambda}{2}$$

This indicates that a bright or dark spot appears each time distance (d) between the two glass sheets is changed by $\lambda/2$.

As **Fig. 29** shows, when glass sheet $P_1$ is inclined relative to the glass sheet $P_2$, the relationship between the interval of the two adjacent fringes x and the angle of inclination $\beta$ is given by the following formula.

$$x=\frac{\frac{\lambda}{2}}{\tan\beta}\fallingdotseq\frac{\lambda}{2\beta}$$

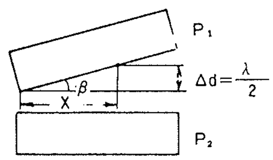



**Fig. 29** Equal thickness interference (non-parallel)

This means that the interval between the fringes is large when the angle $\beta$ is small and vice versa. As shown above, fringes appear where the thickness of the layer of the air is even, and this phenomenon is referred to as *equal thickness interference*. The principle of equal thickness interference is applied to measuring gauge blocks, flatness by optical flats, parallelism by optical parallels, and lens curvature by Newton's rings.

### 4.1.2 Equal inclination interference

Let us consider the question of rays $T_1$ and $T_2$ emitted from a monochromatic light source L, which has a finite area. When $T_1$ and $T_2$ converge on a focal plane after passing through a lens, interference occurs as in the case of equal thickness interference.

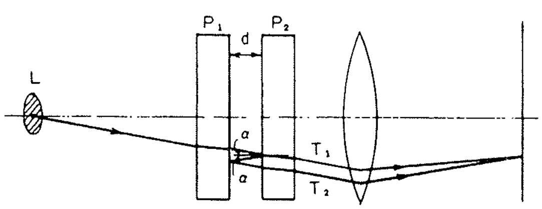



**Fig. 30** Equal inclination interference

The optical path difference $\Delta T$ between $T_1$ and $T_2$ is:

$\Delta T = 2d\cos\alpha+\lambda$

When $\Delta T$ is an even multiple of $\lambda/2$, constructive interference takes place ( $T_1$ and $T_2$ are in phase). Thus the formula that defines the conditions for the appearance of a bright fringe is:

$$d\cos\alpha=n\frac{\lambda}{2}$$

where n = 0,1, 2, ......

The light emitted from all points of the light source parallel to $T_1$ and $T_2$ will converge at the same location as $T_1$ and $T_2$. This then causes interference to take place at that point and the same principle will also apply to rays emitted at other angle. As the optical path difference is determined by the angle $\alpha$, the interference fringes form a number of concentric circles, whose center is on the optical axis of the lens. Because the same interference fringes are caused by light traveling in the same direction, this phenomenon is called *equal inclination interference*. Interferometers applying this principle are easy to fabricate, and for this reason they are often employed in laboratories and in the past were used to measure gauge blocks, but this technique is now rarely used for industrial purposes.

## 4.2 Diffraction of Light

When light waves pass over the edge of a wall or through an opening which is wide compared to the wavelength of the light, they will pass through in parallel, straight lines although there will be a slight bending effect around at the edge. If, however, the opening is very narrow, i.e. around the same order of width as the wavelength, the wavefronts will emerge with a pronounced circular shape. Thus the light spreads to areas where it should not reach according to the law of rectilinear propagation. This phenomenon is known as *diffraction of light*.

### 4.2.1 Slit diffraction

Here, we study the interference of plane waves passing through a slit, when interference takes place at point P, which is at a very long distance from the slit compared to the width of the slit opening. This phenomenon is called *far-field or Fraunhofer diffraction*.

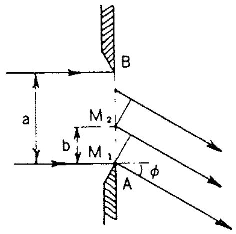



**Fig. 31** Slit diffraction

Let the wavelength of the plane wave be $\lambda$ and the wavefronts assumed to be at right angles to the slit. Assuming that point P is located at an angle $\phi$ with respect to the direction of the incident light, and points $M_1$, $M_2$, .... $M_n$ are taken to be on a line connecting points A and B, dividing the length A-B into sections to satisfy the following formula.

$$M_2P - M_1P = \cdots\cdots = \frac{\lambda}{2}$$

Since the area of each of the divided zones is the same (i.e. the number of beams included in each zone is the same), each of the light waves in one zone will be canceled out by those in the adjacent zone, thus producing a dark band in an angular direction where AB is divided into an even numbers of zones. On the other hand, if light is diffracted in an angular direction where AB is divided into an odd number of zones, a bright band is produced because there is still one remaining zone after the waves in the adjacent zones have canceled each other out. This means that bright and dark fringes appear according to the direction of diffraction. Let the width of a zone equal b, and the width of the slit equal a. Here, the width b is expressed by the following formula.

$$b \sin \phi = \frac{\lambda}{2}$$

Therefore, the dark fringes appear when:

$$2nb = a$$
where $n = 1, 2, \ldots\ldots$

From the above two formulae, it can be seen that the dark fringes appear in the angular direction $\phi$ given by the following formula and the bright fringes appear between the dark fringes.

$$a \sin \phi = n\lambda$$
where $n = 1, 2, \ldots..$

The brightest line is produced in the direction where $\phi = 0$, as none of the light beams that pass through the slit is subjected to destructive interference. The narrower the slit opening, the wider the central bright line and the longer the intervals between the bright fringes. Interference fringes are sometimes observed when carrying out measurement using a microscope.

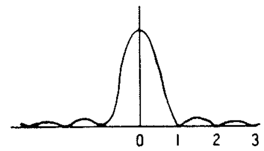



**Fig. 32** Distribution of bright and dark areas produced by diffraction

### 4.2.2 Diffraction gratings

An optical device having a number of close, parallel slits at equal intervals on a plane surface is called a *diffraction grating*, and the distance between adjacent slits is called the *grating constant*. Usually, a sheet of parallel-plane glass with fine, parallel lines lithographed at equal intervals is used, with the transparent sections between the lines serving as slits.

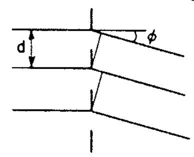



**Fig. 33** Diffraction grating

When plane waves pass through a diffraction grating at right angles to the plane of the grating, and point P is taken at sufficiently long distance from the grating, the optical path difference between any two adjacent rays to reach the point P is given by $d \sin \phi$. Therefore, bright lines are produced in an angular direction $\phi$ that satisfies the following conditions.

$$d \sin \phi = n\lambda$$

where $n=0, 1, 2, \ldots..$

When there is a large number of slits, the bright lines become distinct enough to measure the angle $\phi$ accurately. If the grating constant is known, the wavelength of the light is obtained. Conversely, if the wavelength is known, the grating constant can be calculated. When white light is used with a diffraction grating, a line of white light appears at n=0. In the direction $n \geqq 1$, the location of the bright lines differs depending on the wavelengths, so a spectrum is displayed. This is called a *diffraction spectrum*, and the number n is called the *order of the spectrum.*

### 4.2.3 Resolving power

Even using aberration-free lenses, an object point does not correctly focus on a point image because of the effect of diffraction forming diffractive images. For this reason, there is a certain limitation on the ability of observing two nearby points distinctly when using optical instruments. This limit value is called the *resolving power or resolution* of an optical instrument.

1) The telescope

A classical problem in astronomy is the resolution of double stars or similarly close objects. The telescope aperture, i.e. the rim of the objective, limits the the size of the plane wavefronts accepted from one of the stars and thus acts as a slit, causing far-field diffraction to occur at the aperture.

When two close stars are observed using a telescope, their images A and B formed in accordance with the laws of geometrical optics can be resolved as two different points if they are sufficiently separated so that the primary dark fringe around the image A produced by far-field diffraction does not overlap image B. The two images cannot be distinguished when they are closer than this limit. Taking the effect of far-field diffraction of the telescope aperture into account, the relationship between the diameter of the objective lens 2r and the minimum angular resolution limit is given by the following formula although the proof is not given here.

$$\phi = \frac{0.61\lambda}{r}$$

where $\lambda$ = wavelength of the light

This indicates that the larger the diameter of objective lens, the higher resolving power a telescope possesses.

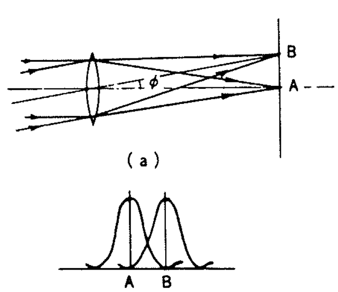



**Fig. 34** The resolving power of telescopes

2) The microscope

Consider a diffraction grating G being placed in the object plane of objective lens L, and assume that diffractive images O, $A_1$, $A_2$ .... are formed on the focal plane F of lens L (**Fig. 35**). The diffractive images on the focal plane F in turn act as a diffraction grating, and form diffractive images of G on the image plane P of the G-L-F optical system. In this case, the relationship between the aperture angle of the objective lens $\phi$ and the grating constant d is expressed by the following formula.

$$d \geqq \frac{\lambda}{\sin\phi}$$

If the grating constant d is taken as the distance between two objects, d expresses the minimum distance of two distinguishable objects. When the refractive index of the object field is n, the wavelength is expressed by $\lambda/n$. Therefore, the above formula can be rewritten as follows.

$$d \geqq \frac{\lambda'}{n\sin\phi}$$

The value n sin $\phi$ is known as the *numerical aperture of the objective lens* and is abbreviated N.A. In a microscope optical system, the larger the numerical aperture of a lens, the higher the resolving power.

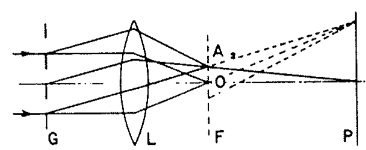



**Fig. 35** The resolving power of a microscope

3) Photographic lens

Let the diameter of the entrance pupil of a lens be equal D and the focal length equal f. The same formula used for a telescope is applicable here, and the resolving power d is given as follows.

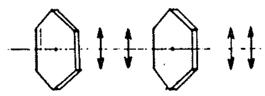
$$d = 1.22\,\lambda\frac{f}{D}$$
$$= 1.22\,\lambda F_{No}$$

where, $F_{No}$. = F number

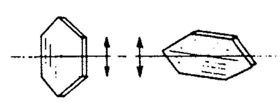
As shown in the formula, the resolving power of a photographic lens is higher when $F_{No}$ is low.

### 4.3 Polarization

When observing light passing through two sliced sheets of a tourmaline crystal, cut parallel to the principal crystal axis and placed facing each other, the light is brightest when the principal crystal axes of the two plates are parallel and dark when they are at right-angles. From this observation, it is known that the light filtered through a tourmaline sheet changes into a different form of light with a characteristic direction of oscillation component. Such light has different intensities in different oscillation directions and is called *polarized light*. The phenomenon caused by the tourmaline crystal is referred to as *linear polarization* or *plane polarization* .



(a) Parallel Nicol Prism



(b) Crossed Nicol Prism

**Fig. 36** Polarization

In the case of plane-polarized light, the *plane of polarization* is defined as the plane containing the direction of propagation and the electric field vector at any point in the radiation field (the electric field vector also determines the plane of polarization of other E.M. waves). A substance producing polarized light is called a *polarizer*, and a detecting device is called an *analyzer*. An instrument which combines a polarizer and an analyzer to determine the effects of substances on polarized light is known as *polariscope*.

Tourmaline is rarely used nowadays, not only due to difficulty in getting large-size crystals, but also because of its high absorption coefficient (about 50%) and the deep color (dark green). A thin film made of powdered rhombic crystals of quinine sulfate periodate, arranged and immobilized so that the crystal axes are parallel to each other, and sandwiched between two glass sheets, produces the same effect as tourmaline. Artificial polarizing plates of this type are now widely used.

We can also have light beams which are mixtures of unpolarized and polarized light. These are said to be *partially polarized.* Most light from everyday sources is partially polarized e.g. sky light, sun light, light from metal filament lamp, but sometimes the polarized component is a small proportion of the total intensity. Natural white light, when reflected from a smooth surface (specular reflector) also becomes partially polarized, with maximum amplitude in a direction perpendicular to the plane of incidence. Light, when refracted, changes into partially polarized light but, in this case, the maximum amplitude occurs in the direction of the plane of incidence. The degree of partial polarization is relative to the angle of incidence. When the reflected light and the refracted light are at right angles, the reflected light becomes completely plane polarized and the refracted light undergoes maximum polarization. This is called *Brewster's law*, and the angle of incidence where this phenomenon occurs is called the *polarizing angle* or *Brewster's angle*. When the refractive index of the medium is n, and the polarizing angle is $\phi$, the following formula is derived from the law of refraction.

$$\sin\phi = n \sin\beta$$

The condition under which the reflected and refracted rays are perpendicular is expressed by the following formula.

$$\left(\frac{\pi}{2} - \phi\right) + \left(\frac{\pi}{2} - \phi\right) + \frac{\pi}{2}$$

This can be rewritten in the form:

$$\beta = \frac{\pi}{2} - \phi$$

Combining this formula with the first one,

$$\sin\phi = n \cos\phi$$

Therefore,

$$\phi = \tan^{-1} n$$

For example, for n = 1.51633 (BK7 spec. glass):

$$\phi \fallingdotseq 56° \, 36'$$

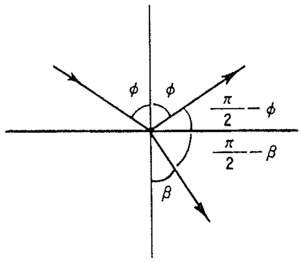


Fig. 37 Polarizing angle

This means that light falling on BK7 glass at an incident angle of 56° 36' will, when reflected, be plane polarized.

In the polariscope shown in **Fig. 38**, when the two polarizers constitute a crossed Nicol prism, light will not pass through the polariscope. However, when a crystal plate is placed between the two polarizers, light can pass through the crossed Nicol prism; generally, if the source generates monochromatic light, the output is comparatively lighter and if the source generates white light, complementary colors are produced at the output and is called *chromatic polarization*. This phenomenon can be explained by the mechanism of linearly polarized light changing its direction of vibration as it propagates through the crystal plate. This principle can be used to observe the crystalline structures of a substance.

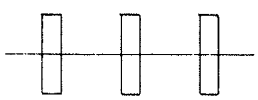



Fig. 38 Polariscope

Transparent, non-crystalline material, when subjected to mechanical stress, also shows the optical characteristics of anisotropic mediums and produces chromatic polarization. Measuring instruments make use of this phenomenon for measuring the internal stress on elastic materials. The polarization phenomenon is also applied to studies of crystals, e.g. to measure the sugar content of a sugar solution.

## 5. LIGHT SOURCES AND PHOTOMETRY

### 5.1 Light Sources

The requirements for a light source to be used in optical measuring instruments are:

1) Homogeneous and constant intensity
2) Small dimensions
3) Sufficient brightness
4) Long service life
5) Low cost

#### 5.1.1 Incandescent lamps

This is the most popular type of electric bulb and is also known as a filament lamp. Those for special use in optical measuring instruments have a flat, densely coiled filament, which is almost square when viewed from the front. As the filament depth is small it can be considered for all practical purposes as a surface source. In order to produce high power bulbs at given low voltage, a thick, short filament is used, conversely, a long, thin filament is used for bulbs with a high voltage rating and the same power consumption. When manufacturing a bulb which produces bright light with a small illuminating surface area, it is necessary to keep the rated voltage low and use a short, thick filament. The electrical specification of bulbs normally used for profile projectors or optical comparators are: 10V-70W, 12V-100W, 12V-150W, etc. Tungsten is used as filament material because of its high melting point and low vaporization rate. To minimize tungsten vaporization, an inert gas (argon) is normally used to fill the bulbs.

When incandescent bulbs are used outside their specified voltage rating, the light characteristics change according to the voltage. As shown in **Table 2**, the power consumption, efficiency and luminous flux increase with a rise in voltage, but the service life is reduced. When bulbs are used at 90% of their rated voltage, their service life is prolonged by approximately three times the normal span. Therefore, it is wise to adjust the brightness (voltage) to meet the conditions as needed.

**Table 2** Voltage vs Service life

| Voltage (%) | Service life (%) |
|---|---|
| 80 | 1500 |
| 90 | 300 |
| 100 | 100 |
| 110 | 30 |
| 120 | 10 |

### 5.1.2 Halogen lamps

Electric bulbs filled with a halogenous gas, such as iodine, bromine, chlorine and fluorine, are called halogen lamps. The addition of a halogen gas to a tungsten lamp can increase the life and efficiency of the lamp. At low temperature locations inside the bulb, the halogen gas combines with the evaporated tungsten to form a tungsten-halogen compound, maintaining transparency in the bulb. The resulting tungsten-halogen compound migrates back to the filament, where at very high temperatures it decomposes and the evaporated tungsten is redeposited on the filament, completing the cycle. This reduces filament carbonization and results in uniform and constant light output throughout the lamp's life.

Of the various types of halogen lamps, those using iodine gas are called *iodine lamps*. Since the volume of an iodine lamp is 1/100 the volume of an equivalent incandescent bulb, the use of this type of lamp can reduce the size of the illuminator used in optical instruments. The most frequently used lamps for optical measuring instruments are *bromine lamps,* with a volume about 1/40 the volume of an equivalent incandescent bulb.

Halogen lamps display similar electrical characteristics to incandescent bulbs. As **Table 3** shows, a lower voltage gives longer service life, for example, if a halogen lamp is used at 90% of the rated voltage, the service life is prolonged up to four times longer.

**Table 3** Voltage vs Service life

| Voltage (%) | Service life (%) |
|---|---|
| 80 | 2000 |
| 90 | 400 |
| 100 | 100 |
| 110 | 35 |
| 120 | 13 |

### 5.1.3 High-pressure mercury vapor lamps

High-pressure mercury vapor lamps are designed to generate light by electrical discharge in a mercury vapor pressurized to a few atms. This kind of lamps' efficiency is typically more than twice that of incandescent lamps.

From practical point of view, mercury lamps have some disadvantages, such as:

(1) After switching on, it takes about 10 minutes for the light to become stable.

(2) Once the light is turned off, it is not possible to immediately restart it.

(3) The price is higher than ordinary lamps, as they need a stabilizer unit. Therefore, they are mainly used for special optical measuring instruments, high-magnification profile projectors, etc.

## 5.2 Photometry

### 5.2.1 Luminous intensity

The amount of radiant energy flowing from a point source per unit time is defined as the *luminous flux* and is measured in lumens (ℓm), which is defined as the luminous flux emitted from a point source having a uniform intensity of 1 cd (candela; see below) within a unit solid angle (1 stenradian).

The luminous flux incident on a small surface perpendicular to a specified direction from the light source, divided by the solid angle (in steradians) that the surface subtends at the light source is called the *luminous intensity* or *light intensity* . The unit of luminous intensity is named the *candela* (cd) and is defined as 1/60 the luminous intensity per square centimeter of a blackbody radiator operating at the freezing temperature of platinum (2045° K). Sometimes, another unit of luminous intensity, "candlepower", is used. For this, a lamp called a Vernon-Harcourt lamp is adopted as a standard. One candlepower (equivalent to 1.018 cd) is defined as 1/10 the horizontal luminous intensity of the lamp, which generates light by burning a mixture of pentane ($C_5H_{12}$) and air at a ratio of 7:20.

### 5.2.2 Illuminance

The density of the luminous flux falling on a surface is called *illuminance*, and the unit of illuminance is the *lux* (ℓx). When luminous flux measuring 1 ℓm falls on surface area measuring 1 m², the illuminance on that area is equal to 1 ℓx.

The solid angle Ω is given by the radius (r) of a sphere and the area of spherical surface S generated by the solid angle, as follows.

$$\Omega = \frac{S}{r^2}$$

Here, the unit solid angle in steradians (sr) is defined as the angle that will subtend a spherical surface area of 1 m² at a distance of 1 m, taking the angle vertex to be at the center of the sphere. As mentioned above, the luminous flux from a point source of 1 cd emitted within 1 sr is 1 ℓm. We can thus say that the illuminance on perpendicular surface at a distance of 1 m from the light source of 1 cd is 1 ℓx.

The illuminance (E ℓx) on a perpendicular surface at a distance r m from a light source of I cd is expressed by the following formula:

$$E = \frac{I}{r^2}$$

The illuminance on an inclined surface at an angle q in respect to the direction of light is, assuming the other terms remain constant, proportional to cos q. The illuminance E in this case can be given as:

$$E = \frac{I}{r^2}\cos\theta$$

This is called *Lambert's law.*
The screen illuminance of profile projectors is in the range 100 - 150 ℓx for low magnification units, and 20 - 50 ℓx for high magnification units. The most suitable illuminance for measurement is between 50 - 100 ℓx.

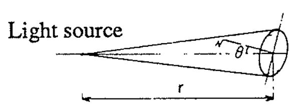



**Fig. 39** Illuminance on an inclined surface

### 5.2.3 Luminance

The concept of luminous intensity only applies when light is emitted from a point source. However, if the surface area of a light source is taken into consideration, the term *luminance* is used. The unit of luminance is the cd/m², which is defined as luminous intensity in a direction perpendicular to the illuminating surface with an area of 1 m² and luminous intensity of 1 cd, when the brightness of the illuminating surface is homogeneous to that direction. The unit cd/m² is expressed by 1 nit, but now the unit of stilb (sb) as defined below, is more frequently used.

$$1\ \text{sb} = 1\ \text{cd/cm}^2 = 10^4\text{cd/m}^2$$

## 6. OPTICAL INSTRUMENTS

### 6.1 Telescope

1) The Galilean telescope
A Galilean telescope consists of a convex objective lens and a concave eyepiece, which are arranged in such a way that the focal planes of the objective and eyepiece coincide at the rear of the eyepiece.

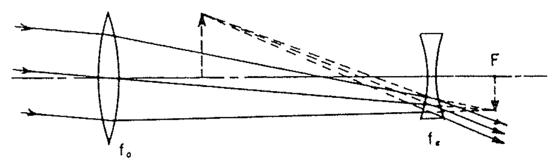



**Fig.40** The Galilean telescope

This type of telescope forms an erect image, and for this reason they are used as terrestrial telescopes, opera glasses, etc., but high magnification is unavailable. The magnification of a telescope is expressed by the following formula, when the focal length of the objective and eyepiece are fo and fe respectively.

$$m = \frac{fo}{fe}$$

2) The Kepler telescope
The objective and eyepiece of the Kepler telescope are both convex lenses, and are arranged in such a way that the focal planes coincide at the front of the eyepiece.

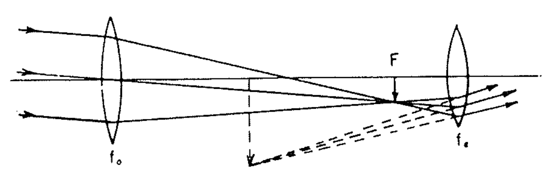
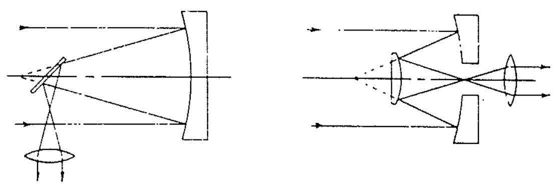



**Fig. 41** The Kepler telescope

This type of telescope is mainly used for astronomical telescopes, because it forms an inverted image, although they can be designed to form an erect image by inserting an erecting lens or an erecting prism between the objective and eyepiece. In this way, they can also be used for terrestrial observations. The magnification is given by the following formula (same as the Galilean telescope).

$$m = \frac{fo}{fe}$$

3) The reflecting telescope
Due to distortion caused by lens aberrations and to increase resolving power, most large, modern astronomical telescopes use mirrors as the objective. The two most common systems are the Newtonian and the Cassegrain systems, both use a parabolic primary mirror as the objective. Presently, the world's largest reflecting telescope, completed in 1974, was built at the Astrophysical Observatory, Academy of Sciences of the USSR, and has an mirror diameter of 6 meters.



Newtonian system    Cassegrain system

**Fig. 42** Reflecting telescope

## 6.2 Microscope

A compound microscope has a convex lens for both the objective and the eyepiece. Letting the magnification of objective lens equal $m_o$ and the eyepiece equal $m_e$, the system magnification is given by the following formula:

$$m = m_o m_e$$

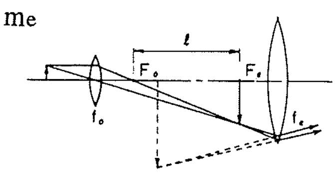



**Fig. 43** The microscope

The magnification of the objective lens is given by the following formula, letting its focal length equal $f_o$ and the distance between its rear focal point and image point equal $\ell$.

$$m_o = \frac{\ell}{f_o}$$

The distance $\ell$ is sometimes called the *optical tube length.* The magnification of an eyepiece is expressed by the following formula, where its focal length is $f_e$ and the shortest distance of distinct vision is D.

$$m_e = \frac{D}{f_e}$$

The shortest distance of distinct vision for the human eye is approximated at 250 mm for basis of calculations.

## 6.3 Photometrical Instruments

### 6.3.1 The measuring microscope

Measuring microscopes are classified into three major categories, although they employ practically the same optical system.

Measuring microscope
- Measuring microscopes for factory use
- Toolmaker's microscopes
- Universal measuring microscopes

The use of a telecentric system (a system whose aperture stop is located at one of the foci of the objective lens), has the advantage of minimizing the effect of focussing errors on measurement results. The objective lens used in measuring microscopes is compensated for distortion, curvature of field and chromatic aberration. The eyepiece incorporates a glass plate, inscribed with outline of screws, gears, etc., in order to increase the efficiency of measurement work.

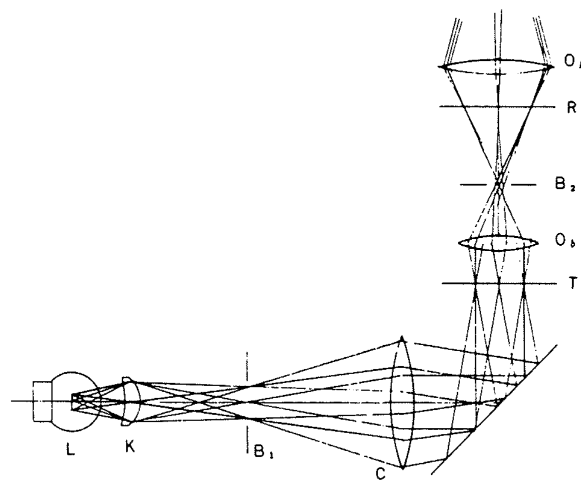



**Fig. 44** The basic optical system found in a measuring microscope

The magnification ratio of a measuring microscopes is generally 10X to 100X. The reason for using this range of magnification is that, theoretically, these values should be adequate for normal use as the naked eye can only discriminate fine details up to the resolution limit value multiplied by the magnification of the microscope. When viewing an object the same size as resolution limit value d is magnified M times to form an image at a distance S from the eye is , the angular aperture ω with respect to the eye is given by the following formula:

$$\omega = \frac{dM}{S} = \frac{M}{S}\,\frac{\lambda}{NA}$$

Therefore,

$$M = \frac{\omega S\,NA}{\lambda}$$

Put the following values into the above formula to obtain the value of M.

$\lambda = 0.00058$ mm (median wavelength of visible light)

S= 250 mm

$\omega = 1'$

Then,

$M \fallingdotseq 130$ NA

This means that magnification is proportional to the value of NA and a suitable value of magnification is about 130 times the NA (numerical aperture). Making some allowances, we may consider that the appropriate magnification will be in the range:

$$300\,NA < M < 700\,NA$$

Any larger magnification will only result in impairing the clarity of the image. The NA of the objective lens is usually in the range 0.07 to 0.15, and, therefore, the required magnification ratio should be:

$$20 < M < 100$$

The standard magnification of a measuring microscope is normally set to 30X, which falls within this working range.

### 6.3.2 Profile projectors

Telecentric optical systems are also used for profile projectors. Similar to measuring microscopes, they are designed in such a way that any focussing errors will have a minimal effect on the measurement results, but the basic optical system is simpler than that of the microscope. The projection lens is compensated for distortion, curvature of field and chromatic aberrations. **Fig. 46** shows the optical system of a profile projector and the way it forms an image.

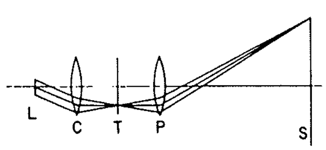



**Fig. 45** The optical system of a profile projector

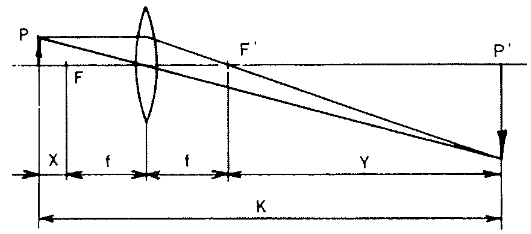



**Fig. 46** The image forming system of a profile projector

The relationships between the value factors in the system are given in the following formulae.

$$xy = f^2$$

$$m = \frac{y}{f} = \frac{f}{x} = \frac{y+f}{f+x}$$

$$K = x + y + 2f$$

Therefore,

$$f = \frac{m}{(m+1)^2}K$$

$$x = \frac{1}{(m+1)^2}K$$

$$y = \frac{m^2}{(m+1)^2}K$$

From the above formula, it can be seen that the larger the magnification ratio, the shorter the distance of the object point, i.e., the distance ($\ell$) from the apex of the objective lens to the focusing position of the object. The distance from the end of the lens frame to the focusing position is called the *working distance* (W.D). A longer working distance provides easy operation. However, the working distance of an aberration-compensated lens is normally shorter than its focal length. To eliminate this problem, some profile projectors include relay lenses in the optical system.

Although this system is very useful in that the working distance is kept constant regardless of magnification ratio, the complicated structure renders the system more expensive.

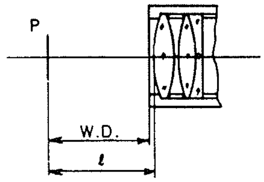



**Fig. 47** Working distance

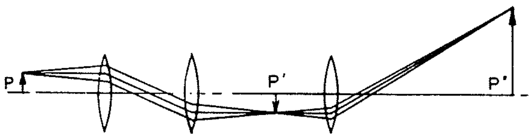



**Fig. 48** Optical system with relay lenses

## 7. GLOSSARY

### 7.1 Terms for Geometrical Optics

1. Optical axis

(1) An imaginary straight line passing through the center of the light source, lenses and diaphragm in an optical system.

(2) An imaginary straight line passing through the center of curvature of curved surfaces used for refraction (reflection) when these surfaces constitute an optical system.

2. Conjugate

The relationship between the object and the image in an optical system.

3. Focal point / Focus

The conjugate point of an object point at an infinite distance. When the object point at an infinite distance is in object space, the focal point is called the *image focal point*; and when the object point is in image space, it is called the *object focal point.*

4. Focal plane

A plane perpendicular to the axis of an optical system and passing through the focal point of the system.

5. Focal length

The distance from the principal point to the image focal point in an optical system.

6. Principal point

A pair of conjugate points on the optical axis, where the lateral magnification is +1. The principal point in object space is called the *object principal point*; the principal point in image space is called the *image principal point.*

7. Principal plane

A plane perpendicular to the optical axis at the principal point.

8. Magnification

The ratio between the size of the object and the size of the image formed in an optical system. There is lateral magnification, longitudinal magnification and angular magnification. The term magnification, when used without specific exposition, generally refers to lateral magnification.

9. Real image

An image that is formed in image space by a pencil of rays that passed through an optical system.

10. Virtual image

An image that is formed in object space by a pencil of rays (e.g. divergent pencil of rays) that have passed through an optical system.

11. Erecting system

An optical system that inverts an image vertically and horizontally.

12. Erecting image

An image whose lateral magnification is a positive value both vertically and horizontally.

13. Inverted image

An image whose lateral magnification is a negative value both vertically and horizontally.

14. Convergent pencil of rays

A pencil of rays that converges into a point.

15. Divergent pencil of rays

A pencil of rays that diverges from a point.

16. Reflection factor

The ratio between the intensity of the reflected ray Ir to that of the incident ray Ii. It is normally expressed as a percentage.

17. Transmission factor

The ratio between the intensity of transmitted ray It to that of the incident ray Ii. It is normally expressed as a percentage.

18. Scattering
A phenomenon in which the direction of light propagation is changed irregularly, when the light collides with a particle or substance.

### 7.2 Terms for Physical Optics

1. Visible ray
Light whose wavelengths are within the 380 nm - 780 nm range and can be sensed by the eye.

2. Ultraviolet radiation
Radiation whose wavelengths are within the 10 nm - 380 nm range.

3. Infrared radiation
Radiation whose wavelengths are longer than 780 nm (up to $1 \times 10^5$ nm).

4. White light
Continuous spectrum light that is perceived as white light by the eye.

5. Monochromatic light
Light whose wavelength is of a specific length.

6. Wavelength
The distance between two points on light wave at $2\pi$ phase difference.

7. Optical path length
A value given by the product of the distance of light propagation $\ell$ and the refractive index n of the medium through which the light propagates.

8. Optical path difference
The difference between two optical paths.

9. Phase difference
The value of the optical path difference expressed by an angle unit.
Phase difference = $\frac{2\pi}{\lambda}$ x optical path difference
where, $\lambda$ = wavelength of light in a vacuum.

10. Fizeau fringes
Interference fringes generated by equal thickness interference.

11. Haidinger rings
Interference fringes with a concentric circle pattern generated by equal inclination interference.

12. Natural light
Light that is not polarized.

13. Double refraction
A phenomenon in which incident light is refracted by anisotropic substance and divided into two separate light waves that have the directions of vibration perpendicular to each other.

### 7.3 Terms for Optical Instrument

1. Principal ray
A ray emitted from an object point off the optical axis that passes through the center of the aperture diaphragm of the lens system.

2. Aperture diaphragm
A diaphragm that limits the pencil of rays into a lens system.

3. Field stop
An opening that limits the field of view of an optical instrument.

4. Telecentric system
An optical system whose aperture stop is located at one of the foci of the objective lens so that the principal ray passes through the focal point.

5. Aperture ratio
The ratio of the entrance pupil diameter (D) to the focal length (f)
Aperture ratio = $\frac{D}{f}$

6. F-number
The reciprocal of aperture ratio.

7. Angular aperture
The angle subtended at an axial object point of an optical system by the diameter of the entrance pupil, or the angle subtended at the image point by the diameter of the exit pupil.

8. Numerical aperture
A measure of the resolving power of a microscope objective and is given by n sin a, when an axial object point in the medium, whose refractive index is n, is subtend by the radius of the entrance pupil to an angle a. Abbreviated N.A.

9. Entrance pupil
The image of the aperture diaphragm formed in object space. In the case of a photographic lens, it is the image of the aperture diaphragm formed by the optical system in front of the aperture diaphragm.

10. Exit pupil
The image of the aperture diaphragm formed in image space. In the case of a photographic lens, it is the image of the aperture diaphragm formed by the optical system behind the aperture diaphragm. The position of the exit pupil of a telescope or microscope is called the *eye point.*

11. Effective aperture
The maximum diameter of a pencil of rays in object space emitted from an axial object point at an infinite distance and passes through an optical system.

12. Flare
A phenomenon of light spreading on an image surface, caused by multiple internal reflections in an optical system.

13. Ghost
Undesirable images formed on the image surface of an optical system that appear off the regular image location (duplicate images).

14. Focusing
Operation to make the illuminated surface of an optical instrument coincide with the conjugate image surface of the object. It also means adjusting the focus to produce a sharp image.

15. Depth of focus
(1) The range of distances off the best focusing point of a photographic lens, within which a clear image can still be obtained.
(2) The range of distances in the axial direction in object space, within which a microscope or measuring projector can form a clear image.

16. Resolution limit
The minimum distance between two points or lines that can be distinguished in an optical system.

17. Resolving power
The ability of an optical instrument such as a telescope, microscope, or human eye, to distinguish two separate points or lines close to each other.

18. Resolution reading
A number indicating how many lines per millimeter are contained in the finest group that can be distinguished on a resolution chart. The resolution chart is device to test resolving power; usually alternate black and white lines of equal width arranged in groups of decreasing line width, identified as the number of line pairs per millimeter.

19. Tangential resolving power
The resolving power measured in respect to alternate black and white lines drawn perpendicular to a line connecting a given off-center point and the center of the image plane.

20. Radial resolving power
The resolving power measured in respect to alternate black and white lines drawn parallel to the line connecting a given off-center point and the center of the image plane.

21. Goniometer eyepiece
An objective lens that has a rotatable reticle for measuring angles.

22. Template eyepiece
An eyepiece that has an overlay chart printed with patterns or shapes.

### 7.4 Other Related Terms

1. Laser
A system or a device used to generate a very intense, spectrally pure, and spatially coherent beam of light, making use of stimulated emission from excited atoms in an optical resonator. There are many laser classifications: solid lasers, gas lasers and semiconductor lasers, etc. according to the type of materials used to construct the laser. The laser most commonly utilized for measuring purposes is the He-Ne gas laser.

2. Stimulated emission
A phenomenon in which atoms or molecules placed in an electromagnetic field, whose wave frequency is almost equal to the natural emission frequency of the substance, emit radiation proportional in strength to the energy density of the electromagnetic field. Stimulated emission does not take place unless the atoms or molecules are in a state of inverted population.

3. Holography
A technique for recording, and later reconstructing the amplitude and phase distributions of a wave disturbance. In optical image formation, the technique is accomplished by recording on a photographic plate the pattern of interference between coherent light reflected from the object, and light that comes directly from the source.

4. Hologram
A record, on a dry plate or on other photographic material, of the interference patterns generated by coherent light reflected from an object interfering with light coming directly from the source, in the holography device.

5. Moiré effect
The effect whereby, when one family of curves having a regular pattern is superimposed on another so that the curves cross at an angle less than 45°, a new family of curves called *moiré* appear which pass through the intersections of the original curves.

– MEMO –

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